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## FIRMS DECISION MAKING PROCESS IN AN EVOLUTIONARY MODEL OF INDUSTRIAL DYNAMICS

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Evolutionary model of industrial dynamics, presented in this paper, can be classified as Schumpeterian one. The model describes the behaviour of a number of competing firms producing functionally equivalent products. Each firm tries to improve its position in the industry and in the market by introducing innovations in order to minimize the unit costs of production, maximize the productivity of capital, and maximize the competitiveness of its products on the market. The problem how decisions are made seems to be crucial for relevant modelling of socio-economic processes. The main aim of the simulations presented in the second part of the paper is to show how fluctuations and discontinuities occurs in economic processes due to boundedly rational decisions of competing firms. It is shown how fluctuation of 3–6 years and of 10 years periodicity can occur in an industry development because of firms' bounded rationality. Long waves of development of 50–60 years period (Kondratieff cycles) occur in the model because of radical innovation emergence at the maturity phase of an 'old' technology.

One of discussed problem within a sphere of socio-economic modeling is a mode of analyzing: should we analyze these precesses as continuous or as a discrete ones? It seems that there is no final answer for this controversy. Generally speaking we can say that economists applying mechanical metaphors use well know analytical tools based on difference equation approach (borrowed from physics) and the economists using biological metaphors are inclined to use discrete time and discrete simulation. The author belongs to the second group of economists. It can be said that there are some fundamental differences between the natural processes of material (physical) world and the social processes, so that the direct applications of formal approaches of physicists to describe the processes of social tissue face fundamental problems. One of the important feature of social processes is that judgements of value and free will of human beings lead to conscious and subconscious processes of choice. Decision-making processes observed only in the social sphere of life are made on the

basis of our expectations, and our scale of values. Decisions process is from its nature a discrete process. Humans decisions are based on some qualitative evaluations of the past development and actual state of the art of our socio-economic environment. Therefore the models of socio-economic processes are a mixture of qualitative and quantitative variables as well as recurrent and non-recurrent events [3].

Presented model can be classified as Schumpeterian one and is rooted in the tradition established in the second part of the XXth century by Nelson and Winter [6]. Last decades witness a vigorous research process toward searching for alternative models to the orthodox economics. Different authors propose different approaches, let we mention only selected ones proposed by Silverberg *et al.* [11], Silverberg and Lehnert [12], Silverberg and Verspagen [13, 14]. Bruckner, *et al.* [1] and Saviotti and Mani [7]; who proposed a model of generalized Lotka–Volterra type based on replicator dynamic approach to describe technological evolution).

A distinguished feature of Silverberg and Verspagen models is that technological progress is embedded in vintage capital. In the model presented in [11] firms are self-financing using their cash and liquid interest bearing reserves. An investments plan of each firm is based on its financial strength. Textbooks' notions of 'demand' and 'supply' are not present in the model. Instead of it, firms' behaviour is placed in more realistic spaces of orders, order backlog, delivery delay, rate of capacity utilization, shipment, etc. The current level of production is constrained by a firm's maximum capacity and the production of each firm depends on prime unit labour cost (i.e., an average over all capital vintages). Similar idea that firms rely on rather simple rules of thumb or routines rather than explicit optimization procedures is applied in models developed by Silverberg, Lehnert and Verspagen [12]–[14].

Different approach, focused mainly on the innovation processes, is proposed by Bruckner, *et al.* [1]. They start from observation of physicists that "relationship between micro- and macro-level descriptions become important and led to questions of fundamental relevance" and that "relatively independent of the nature of the sub-systems mainly the manner of their coordination is important for the demonstration of the well-known macroscopic phenomena of spontaneous structure formation". The authors apply general  $n$ -dimensional birth-death transition model to describe technological development. It is assumed that firms consist of different plants using different technologies. In a general term, the system is described by a number of fields (which in a case of technological process are production units used by different firms applying specific technology  $i$ . Elementary process of self-reproduction, spontaneous generation, self-amplification (i.e. non-linear self-reproduction), sponsoring, error reproduction, cooperative and non-cooperative exchange, spontaneous decline and self-inhibition are a base theoretical concept of the model. Development of the system is described by a Master Equation system defining probability distribution of technologies.

Model presented in this paper apply evolutionary metaphors. Due to space limitations, the presentation of the model will be confined to a general description without going into the mathematical details. Description of the model (presented in

the next section) is focussed on the decision making process undertaken by firms in discrete moments. The model is described in detail in [4, 5]. It seems that the problem how decisions are made is crucial for relevant modelling of socio-economic processes. In fact the mode of development of socio-economic processes mainly depends on a way decisions are made. In the second section selected number of situational experiments are presented. The main aim of these simulations is to show how fluctuations and discontinuities occurs in economic processes due to boundedly rational decisions of competing firms.

### **1. The Evolutionary Model of Industrial Dynamics**

The model describes the behaviour of a number of competing firms producing functionally equivalent products. The decisions of a firm relating to investment, price, profit, etc. are based on the firm's evaluation of behaviour of other, competing firms, and the expected response of the market. The firm's knowledge of the market and knowledge of the future behaviour of competitors is limited and uncertain. Firms' decisions can thus only be suboptimal. The decisions are taken simultaneously and independently by all firms at the beginning of each period (e.g. once a year or a quarter). After the decisions are made the firms undertake production and put the products on the market. The products are evaluated by the market, and the quantities of different firms' products sold in the market depend on the relative prices, the relative value of products' characteristics and the level of saturation of the market. In the long run, a preference for better products, i.e. those with a lower price and better characteristics, prevails.

Each firm tries to improve its position in the industry and in the market by introducing innovations in order to minimize the unit costs of production, maximize the productivity of capital, and maximize the competitiveness of its products on the market. The product's price depends on the current technology of the firm, on market structure and on the assumed level of production to be sold on the market. Price and volume of production are established in an interactive way to fulfil the firms objectives (i.e., to keep relatively high profits in the near future and to assure further development in the long run). In managing innovation, each firm takes into account all economic constraints, as they emerge during the firm's development. It thus frequently occurs that economic constraints prevent a prosperous invention from being put into practice.

One of the distinguished features of the model is the coupling of technological development and economic processes. Current investment capacity is taken into account by each firm in the decision making process. Success of each firm in the search for innovation depends not only on R&D funds spent by each firm to search for innovation, but also on the extent to which firms make private knowledge public. Making the private knowledge of a firm public can in some cases speed up industrial development, but also diminishes a firm's incentives to spend more funds on R&D projects.

There are two ways in which firms search for inventions: autonomous, in-house research, and imitation of competitors. Public knowledge allows not only for imitation of competitors, but may also concern the research process. From all inventions only a small fraction is selected to actually be used. Innovation may modernize current production but can also initiate new, radical way of production, i.e. by introducing essentially new technology. In general, each innovation may reduce unit costs, increase the productivity of capital, and improve product performance. However, it frequently happens that improvement of one factor is accompanied by deterioration of the two other. Firms therefore face the problem of balancing positive and negative factors of each invention. An invention will only become an innovation if the positive factors prevail.

Simulation of industry development is done in discrete time in four steps:

- (1) Search for innovation (i.e., search for new sets of routines which potentially may replace the old set currently employed by a firm).
- (2) Firms' decision making process (calculation and comparison of investment, production, net income, profit, and some other characteristics of development which may be attained by employing the old and the new sets of routines).
- (3) Entry of new firms.
- (4) Selling process (market evaluation of the offered pool of products; calculation of firms' characteristics: production sold, shares in global production and global sales, total profits, profit rates, research funds, etc).

### ***The search for innovation***

According to the tradition established by Schumpeter (see e.g. [6]), we use the term 'routine' to name the basic unit of the hereditary information of a firm. The set of routines applied by the firm is one of the basic characteristics describing it. In order to improve its position in the industry and in the market, each firm searches for new routines and new combinations of routines to reduce the unit costs of production, increase the productivity of capital, and improve the competitiveness of its products in the market. Nelson and Winter [6, p. 14] define routines as "regular and predictable behavioral patterns of firms" and include in this term such characteristics as "technical routines for producing things . . . procedures of hiring and firing, ordering new inventory, stepping up production of items in high demand, policies regarding investment, research and development, advertising, business strategies about product diversification and overseas investment". Productivity of capital, unit costs of production, and characteristics of products manufactured by a firm depend on the routines employed by the firm (examples of the product characteristics are reliability, convenience, lifetime, safety of use, cost of use, quality and aesthetic value). We may speak about the existence of two spaces: the space of routines and the space of product characteristics.

We assume that at time  $t$  a firm is characterized by a set of routines actually employed by the firm. There are two types of routines: *active*, that is, routines

employed by this firm in its everyday practice, and *latent*, that is, routines which are stored by a firm but not actually applied. Latent routines may be included in the active set of routines at a future time. The set of routines is divided into separate subsets, called segments, consisting of similar routines employed by the firm in different domains of the firm's activity. Examples are segments relating to productive activity, managerial and organizational activity, marketing, and so on. In each segment, either active or latent routines may exist. The set of routines employed by a firm may evolve. There are four basic mechanisms for generating new sets of routines, namely: *mutation*, *recombination*, *transition* and *transposition*. The probability of discovering a new routine (*mutation*) depends on the research funds allocated by the firm for autonomous research, that is, in-house development. It is assumed that routines mutate independently of each other. The scope of mutation also depends on funds allocated for in-house development.

The firm may also allocate some funds for gaining knowledge from other competing firms and try to imitate some routines employed by competitors (*recombination*) — for example, each firm can gain knowledge about a domain of activity of another firm by licensing. A single routine may be transmitted (*transition*) with some probability from firm to firm. It is assumed that after transition a routine belongs to the subset of latent routines. At any time a random *transposition* of a latent routine to the subset of active routines may occur.

In general, the probability of transposition of a routine for any firm is rather small. But randomly, from time to time, the value of this probability may abruptly increase and very active processes of search for a new combination of routines are observed. This phenomenon is called *recrudescence*. Recrudescence is viewed as an intrinsic ability of a firm's research staff to search for original, radical innovations by employing daring, sometimes apparently insane, ideas. This ability is connected mainly with the personalities of the researchers and random factors play an essential role in the search for innovations by recrudescence, so the probability of recrudescence is not related to R&D funds allocated by a firm to 'normal' research.

As a rule, mutation, recombination and transposition on a normal level (that is, with low probabilities in long periods) are responsible for small improvements and, during the short periods of recrudescence, for the emergence of radical innovations.

### ***Firm's decisions***

It seems that one of the crucial problems of contemporary economics is to understand the process of decision-making. Herbert Simon states that "the dynamics of the economic system depends critically on just how economic agents go about making their decisions, and no way has been found for discovering how they do this that avoids direct inquiry and observations of the process" [10, p. 38].

The decision making procedure is presented below. Each firm for given price  $p_i(t)$  evaluates the production, investment, expected income and profit in succeeding periods of time.

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(a) *Calculation of the product competitiveness  $c_i(t)$*

Two kinds of product competitiveness are distinguished: technical competitiveness and overall competitiveness (or simply competitiveness). The technical competitiveness reflects the quality of technical performance of the product on the market, and depends directly on the values of the product's technical characteristics, such as reliability, convenience, lifespan, safety of use, cost of use, quality and aestheticism. The overall competitiveness describes product attractiveness on the market and depends on technical competitiveness and the product price. Competitiveness, as a measure of attractiveness of a product, grows with a reduction in its price and an improved technical performance. It is assumed that a product competitiveness at a price  $p_i(t)$  is equal to

$$c(p_i(t)) = \frac{q}{(p_i(t))^\alpha}, \quad (1)$$

where  $q$  is the technical competitiveness,  $\alpha$  the elasticity of price in the competitiveness;  $\alpha$  is a characteristic of the market and describes the sensitivity of the market to price fluctuations. Let us denote by  $c_i(t)$  the competitiveness of products of firm  $i$  at time  $t$ , that is,  $c_i(t) = c(p_i(t))$ .

(b) *Estimation of the average price and average competitiveness*

It may be said, without much exaggeration, that all man's decisions are made on the basis of his expectations, but as Herbert Simon asserts: "economists do not disagree about many things, but they disagree about a few crucial things, in particular, how people form expectations" [10, p. 504]. It is rational to assume that, in general, a firm knows nothing about current and future decisions of competitors. It is assumed that decisions of any firm are made independently on the basis of its expectations of what other firms (competitors) will decide. The simplest assumption is that next time the competitors will behave in a similar way as in the past. Therefore the firm  $i$  estimates that in the succeeding period  $(t, t + 1)$  the average price will be equal to

$$p^e(t) = p^p(t)(1 - f_i(t - 1)) + p_i(t)f_i(t - 1). \quad (2)$$

Similarly, the average competitiveness is expected to be equal to

$$c^e(t) = c^p(t)(1 - f_i(t - 1)) + c_i(t)f_i(t - 1), \quad (3)$$

where  $f_i(t - 1)$  is the market share of firm  $i$  at the previous instant, and  $p^p(t)$  and  $c^p(t)$  are trend values of average price and average competitiveness, respectively. It is assumed that prediction of the trend values  $p^p(t)$  and  $c^p(t)$  is made outside the industry and that these values are known to all firms.

Equations (2) and (3) enable us to model diversified situations faced by different firms, for example, the weight of a small firm to form the average price is much smaller than that of a large firm. So, small firms are, in general, 'price takers' in the sense that they assume that the future average price will be very close to the

trend value, and vice versa, large firms play, in general, the role of ‘price leaders’ or ‘price makers’ so their weight in the formation of the future average price is much more significant.

(c) *Estimation of the global production*

After estimating the average price of all products on the market, the global production sold on the market, that is, the global demand  $Q^d(t)$ , can be estimated. It is assumed that all firms know the demand function,

$$Q^d(t) = \frac{M(t)}{p^e(t)}, \quad (4)$$

where  $M(t)$  is an amount of money which the market is inclined to spend to buy products at an average price  $p^e(t)$ . It is assumed that

$$M(t) = N \exp(\gamma t) (p^e(t))^\beta, \quad (5)$$

where  $N$  is a parameter characterizing the initial market size,  $\gamma$  the growth rate of the market size, and  $\beta$  the elasticity of the average price. The consumption theory and results of empirical research show that almost all price elasticities in demand functions are negative: for primary needs (for example, food, clothing) the elasticities are between 0 and  $-1$ , those of secondary (or ‘luxury’) needs are below  $-1$ . So, it may be expected that for commodities fulfilling primary needs  $\beta$  is greater than zero and smaller than one and for commodities fulfilling higher-order needs  $\beta$  is smaller than zero.

(d) *Estimation of the market share of firm  $i$*

After estimation of the average competitiveness of all products offered for sale on the market and perceiving the competitiveness of its own products, firm  $i$  may try to estimate its future market share. The share of firm  $i$  in period  $(t, t+1)$  is equal to

$$f_i(t) = f_i(t-1) \frac{c_i(t)}{c^e(t)}. \quad (6)$$

It means that the share of firm  $i$  increases if the competitiveness of its products is greater than the average competitiveness of all products offered for sale on the market and declines if the competitiveness is smaller than the average competitiveness.

(e) *Estimation of the production of firm  $i$*

Having the expected share and the expected size of the market, firm  $i$  is able to estimate the quantity of production to be accepted by the market (i.e., the supply of production of firm  $i$ ) on the basis of the simple equation,

$$Q_i^s(t) = f_i(t) Q^d(t). \quad (7)$$

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The capital needed to get output  $Q_i^s(t)$  is equal to

$$K_i(t) = Q_i^s(t)/A. \quad (8)$$

$A$  in the above equation is the productivity of capital.

If the required growth of the capital of firm  $i$  is greater than the investment capability of firm  $i$ , then it is assumed that the capital of firm  $i$  at time  $t$  is equal to the sum of the investment capability and the capital at  $t - 1$ , minus the capital physical depreciation (the amortization). For the capital calculated in such a way, the production  $Q_i^s(t)$  is recalculated as

$$Q_i^s(t) = K_i(t)A. \quad (9)$$

(f) *Estimation of the expected income and profit*

The last step in the decision-making procedure is calculation of the expected income and profit of firm  $i$ , which are equal to

$$\Gamma_i = Q_i^s(t)(p_i(t) - V\nu(Q_i^s(t)) - \eta), \quad (10)$$

$$\Pi_i = \Gamma_i - K_i(t)(\rho + \delta), \quad (11)$$

where  $\Gamma_i$  is the expected income of firm  $i$  at time  $t + 1$ ,  $\Pi_i$  is the expected profit of firm  $i$  at time  $t + 1$ ,  $Q_i^s(t)$  the output (supply) of firm  $i$ ,  $V$  the unit production cost (because there is no innovation,  $V$  is constant and uniform for all firms during the simulation),  $\nu(Q_i^s)$  is the factor of unit production cost as a function of a scale of production (economies of scale),  $\eta$  is the constant production cost,  $K_i(t)$  the capital needed to obtain the output  $Q_i^s(t)$ ,  $\rho$  the normal rate of return and  $\delta$  the physical capital depreciation rate (the amortization).

For a given price  $p_i(t)$  the expansionary investment, the production in the next year, and expected profit and income are calculated by applying the procedure presented above. The problem to be discussed is the way of setting the product price  $p_i(t)$ . It is assumed that a firm takes into account its investment capabilities and estimates the values of an objective function for different prices of its products. The price for which the objective function reaches the maximum value is chosen by a firm as the price of its products. It is not a maximization in the strict sense. The estimation of values of the objective function is not perfect and is made for the next year only; so this is not a global optimization made once and for all but firms apply this rule from year to year.

Different price-setting procedures (based on different objective functions and the markup rules) have been scrutinized, the results of which are presented in [4]. The results suggest that firms apply the following objective function:

$$O_1(t+1) = (1 - F_i) \frac{\Gamma_i(t+1)}{\Gamma(t)} + F_i \frac{Q_i^s(t+1)}{QS(t)}, \quad (12)$$

$$F_i = a_4 \exp\left(-a_5 \frac{Q_i^s(t+1)}{QS(t)}\right),$$



where  $F_i$  is the magnitude coefficient (with values between 0 and 1),  $Q_i^s$  the supply production of firm  $i$  in year  $t + 1$ ,  $\Gamma_i$  the expected income of firm  $i$  at  $t + 1$  (defined by Eq. (10)),  $QS$  is the global production of the industry in year  $t$  and  $\Gamma$  the global net income of all firms in year  $t$ .  $\Gamma(t)$  and  $QS(t)$  play the role of constants in Eq. (12) and ensure that the values of both terms in this equation are of the same order. The function  $O_1$  expresses short- and long-term thinking of firms during the decision-making process (the first and second terms in Eq. (12), respectively). The plausible values of the parameters are  $a_4 = 1$  and  $a_5 = 5$ ; it means that the long-term thinking is much more important for the firms' survival and that the firms apply flexible strategy, that is, the relative importance of short- and long-term components changes in the course of firms' development (the long-term one is much more important for small firms than for the big ones).

The decision-making procedure presented above with the search for the 'optimal' price-setting procedure based on the objective concept constructs a formal scheme for finding the proper value of the price. I treat this scheme as an approximation (abstraction) of what is done by real decision-makers. They, of course, do not make such calculations from year to year, they rather think in the routine mode: "My decisions ought to provide for the future prospects of the firm and also should allow income (or profit) to be maintained at some relatively high level". Decisions on the future level of production and the future product price depend on the actual investment capabilities of the firm.

### ***Products competitiveness on the market***

The productivity of capital, variable costs of production and product characteristics are the functions of routines employed by a firm (see Fig. 1). Each routine has multiple, pleiotropic effects, that is, may affect many characteristics of products, as well as productivity, and the variable costs of production. Similarly, the productivity of capital, unit costs of production and each characteristic of the product can be function of a number of routines (polygenity). We assume that the transformation of the set of routines into the set of product characteristics is described by  $m$  functions  $F_d$ ,

$$z_d = F_d(r), \quad d = 1, 2, 3, \dots, m, \quad (13)$$

where  $z_d$  is the value of characteristic  $d$ ,  $m$  the number of product characteristics, and  $r$  the set of routines. It is assumed also that the productivity of capital  $A(r)$  and the unit cost of production  $V(r)$  are also functions of firm's routines, where these functions are not firm specific and have the same form for all firms.

Attractiveness of the product on the market depends on the values of the product characteristics and its price. The competitiveness of products with characteristics  $z$  and price  $p$  is equal to

$$c(p, z) = \frac{q(z)}{p^\alpha}, \quad z = (z_1, z_2, z_3, \dots, z_m), \quad (14)$$

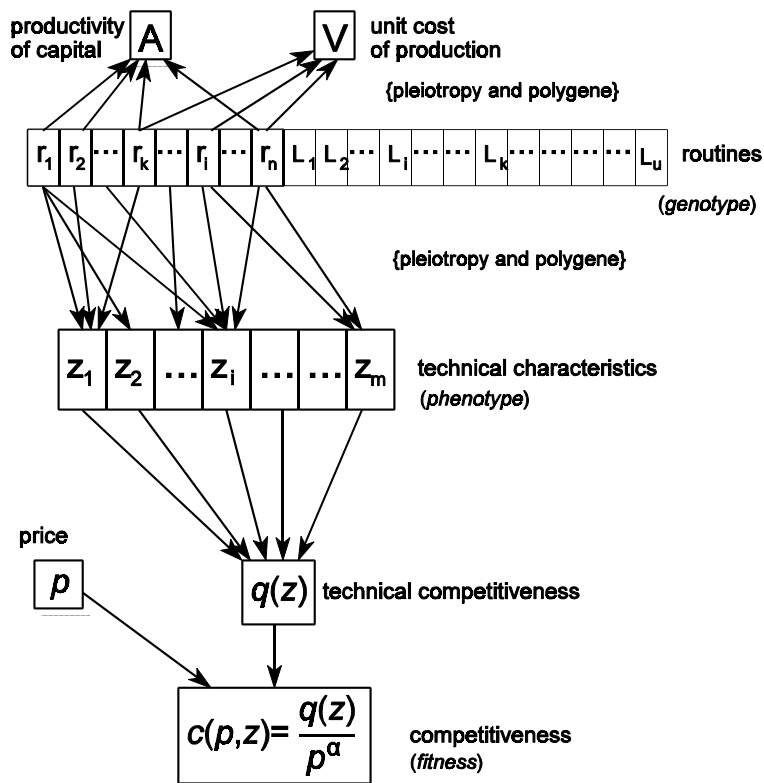


Fig. 1. From routines to competitiveness, productivity of capital and unit cost of production.

where  $q(z)$  is the technical competitiveness,  $z$  a vector of product characteristics, and  $\alpha$  price elasticity.

In the presence of innovation, technical competitiveness varies according to the modification of routines made by each firm, or because of introducing essentially new routines. Technical competitiveness is an explicit function of product characteristics. As explained above, each routine does not influence the product's performance directly, but only indirectly through the influence on its characteristics. We assume the existence of a function  $q$  enabling calculation of technical competitiveness of products manufactured by different firms. We say that  $q$  describes the adaptive landscape in the space of product characteristics. In general, this function depends also on some external factors, varies in time, and is the result of co-evolution of many related industries. The shape of the adaptive landscape is dynamic, with many adaptive peaks of varying altitudes. In the course of time some adaptive peaks lose their relative importance, others become higher.

Due to the ongoing search process, at any moment each firm may find a number of alternative sets of routines. For each alternative set of routines the price, production, investment (including the modernization investment), and values of objective

function are calculated. The decision of firm  $i$  on modernization depends on the expected value of the firm's objective function and its investment capability.

All products manufactured by the entrants and the firms existing in the previous period are put on the market and all other decisions are left to buyers; these decisions primarily depend on the relative values of competitiveness of all products offered, but quantities of products of each firm offered for sale are also taken into account. Similar as in the decision making procedure, the global demand  $Q^d(t)$  for products potentially sold on a market is equal to

$$Q^d(t) = \frac{N \exp(\gamma t) (p(t))^\beta}{p(t)} = N \exp(\gamma t) (p(t))^{\beta-1}, \quad (15)$$

where  $N$  is a parameter characterizing the initial market size,  $\gamma$  the growth rate of the market, and  $\beta$  the (average) price elasticity. The average price of all products offered for sale on the market is equal to

$$p(t) = \sum_i p_i(t) \frac{Q_i^s(t)}{Q^s(t)}, \quad (16)$$

where  $Q^s(t)$  is global supply and is equal to

$$Q^s(t) = \sum_i Q_i^s(t), \quad (17)$$

Global production sold on the market is equal to the smaller value of demand  $Q^d(t)$  and supply  $Q^s(t)$ ,

$$QS(t) = \min\{Q^d(t), Q^s(t)\}. \quad (18)$$

The selection equation describing competition among firms (products) in the market has the following form ( $f_i$  is the market share of products manufactured by firm  $i$ ):

$$f_i(t) = f_i(t-1) \frac{c_i(t)}{c(t)}, \quad (19)$$

where  $c(t)$  is the average competitiveness of products offered for sale,

$$c(t) = \sum_i f_i(t-1) c_i(t). \quad (20)$$

Finally, the quantity of products potentially sold by firm  $i$  (i.e., the demand for products of firm  $i$ ) is equal to

$$Q_i^d(t) = QS(t) f_i(t). \quad (21)$$

The above equations are valid if the production offered by the firms exactly fits the demand of the market. This is a very rare situation and therefore these equations have to be adjusted to states of discrepancy between global demand and global production, and discrepancy between the demand for products of a specific firm and the production offered by this firm. The details of this adjustment process is presented in [5].

## 2. Simulation

The first series of experiment aim to show how fluctuations occur in our model because of the limited firms' computational ability (bounded rationality). It is not denied that monetary factors and innovations play an essential role in fluctuations of economic processes but here I want to point out that the primary factor causing fluctuations ought to be sought in the limited computational ability of man, and related to this natural human proneness to make errors, lapses, fallacies, and so on.

Reason dictates man's actions and from this point of view man may be called a rational being. Rethinking human development from an evolutionary and historical perspective supports the view that man's actions are directed towards the search for a state of affairs that suits him (her) better. But as Herbert Simon observed: "The capacity of the human mind for formulating and solving complex problems is very small compared to the size of the problems whose solution is required for objectively rational behaviour in the world — or even for a reasonable approximation to such objective rationality" [9]. It seems almost impossible that human beings are able to make rational decisions under severe time constraints, huge numbers of variables, and a vast volume of information to be considered in almost every life situation. Human beings manage in such complex situations by considering only a small part of the complexity, making simplifications and idealizations of life situations. To proceed with these complex problems each of us builds a highly simplified mental model of the world. In the end our decisions are made in terms of that model [10, p. 34]. To describe our cognitive situation Simon advanced the hypothesis of bounded rationality [9].

Although in a very stylized form, the concept of bounded rationality is incorporated into our model. Through controlling some parameters of the decision-making procedure we are able to imitate diversified levels of skill ('knowledge and computing power') of the firms to make correct evaluations of investment, price, profit, and so on.

In the decision-making procedure the price, investment, profit and production are established by applying some local optimization procedure. The decision-maker chooses first of all a set of crucial (primary) variables influencing the objective and on the basis of which it is possible to estimate all other characteristics of the economic process (for example, the product price plays the role of the primary variable in our decision-making procedure). Next, variability of the primary variables is assumed and within the domain defined by the variability scope an optimal decision is sought. In principle, it is not possible to present the analytical form of the objective as a function of the primary variables. Therefore, there is no possibility of calculating (estimating) the objective's derivatives and directly determining the optimum (in which the derivatives are equal to zero). At best it is possible to calculate the values of the objective for discrete sets of values of the primary variables. The decision-maker makes such calculations for a finite number of values of the primary variables. The number of such trials depends directly on the computational ability of the decision-maker.

From all the trials the best value is chosen and the values of the primary variables for which the objective reaches maximum (or minimum) are assumed by the decision-maker as his (her) final decision. The distance of that decision from the objectively optimal decision depends directly on the number of trials and the way of choosing the successive values of primary variables (that is, on the optimization algorithm). Something similar is done by the firms in our model. The price is the only primary variable (all others, such as investment and production, are an outcome of the price — as proposed in the decision procedure). The scope of variability of the price is controlled by the model parameter ( $\lambda$ ). The scope of search for the optimal price (that is, the minimum and maximum of the price) depends on the actual value of the firm's product price, namely we assume that  $\text{Min } P_i = p_i/\lambda$ , and  $\text{Max } P_i = p_i\lambda$  (where  $p_i$  is the actual product price of firm  $i$ ). To make the search for optimal price effective, one of the best algorithms of single variable optimization was chosen, namely, the so-called golden division algorithm. Making  $L$  trials the firm is able to reduce the initial scope of search ( $\text{Min } P, \text{Max } P$ ) about  $(1.62)^L$  times. It means that after making, for example, 25 trials, the distance to the optimal price is not greater than  $(\text{Max } P - \text{Min } P)/103,680$ , that is, about  $10^{-5}$  of the initial price range; after making ten trials the reduction is only by a factor of 76 (i.e., around 1.3%).

By assuming different values of the number of trials ( $L$ ) in the optimization algorithm and the price scope of search ( $\lambda$ ) we are able to control the level of the firms' computational ability (computability); the larger the number of trials and the smaller the scope of search for the optimal price, the greater the firm's computability, that is, the firm's decisions may be closer to the optimal ones. Thanks to this property of the optimization algorithm we are able to simulate the influences of bounded rationality on the model's behaviour. We correlate the firm's computability with bounded rationality, and we use the values of the number of trials ( $L$ ) and the price scope of search ( $\lambda$ ) as a measure of the firm's rationality.

As an example, the fluctuations of profit/capital ratio are presented in Fig. 2. If computability is high, the firms are able to find optimal, or very near to optimal, decisions, and the industry moves steadily to the equilibrium state. In the case of poor computability, firms' decisions deviate from the optimal ones, which causes fluctuations in the industry behaviour. Great diversity in the modes of industry development due to different levels of firms' computability is observed. It is impossible to show all types of fluctuations, and therefore only a selection of the simulation results is presented, being only a small part of the observed diversity in the model's behaviour. For relatively high firms' computability (for example, for  $L = 10, \lambda = 2.0$ ) the fluctuations are not significant, and the industry reaches an almost stable equilibrium with the profit around 0.2% (for 'perfect computability' the equilibrium profit is equal to zero). When we reduce the firms' computability the amplitude of the fluctuations rises significantly. For very poor computability ( $L = 7, \lambda = 20$ ) the amplitude fluctuates from 5% to 7%.

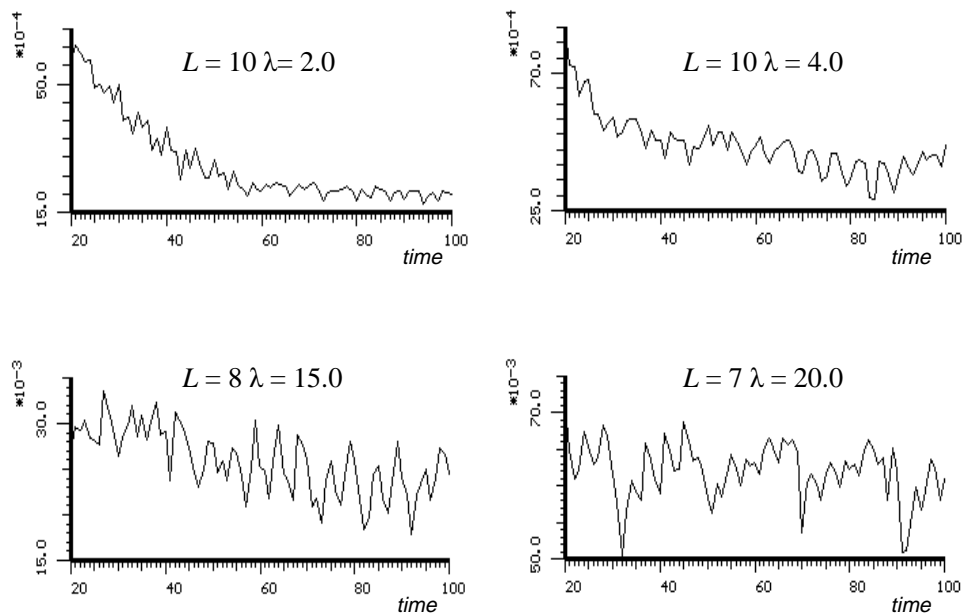


Fig. 2. Profit to capital ratio for the selected values of firms' computability.

It seems obvious that for pure competition we observe an increase in profit due to decreasing firms' computability. The equilibrium profit for high computability (perfect knowledge and infinite computational power) for pure competition is equal to zero. For small computability the firms make their evaluations of the objective values for a limited number of trials. As may be expected, the values of the firm's objective and profit in all trials are greatly diversified, and very rarely the best trial is close to the maximal value of the objective. In the whole set of trials there are cases with positive and negative profits, and it seems natural that the firm chooses the case (trial) which is closest to the optimum and yielding the positive profit. As we may expect, the distance of the best trial from the optimal decision is the farther the poorer the firms' computability.

One may say that making poor estimations is a profitable strategy in the case of pure competitive industry, but if a firm chooses the strategy of making higher 'intentional error estimations' (to gain higher profit), then the prices of its products are also higher. The firm may achieve short-term positive profit, but because of higher prices the competitiveness of its products is smaller and as a direct consequence its market share will drop; in the end the firm will be eliminated from the market. The competition process and free entry of firms ensure that the quality of estimations will be kept at the lowest possible level, which may be called a natural level.

Spectral analysis of the periods of fluctuations does not enable us to find any regular relationship between the period of fluctuations and the level of computability.

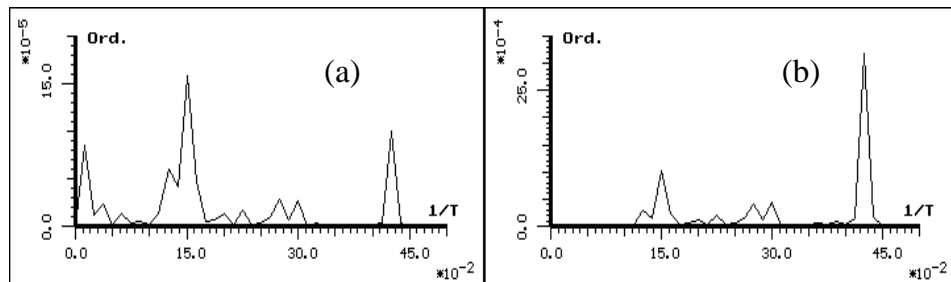


Fig. 3. (a) Spectral density of profit/capital and (b) investment/capital ratios.

Great diversity of the modes of fluctuation is observed in our simulations and even small deviations of computability lead to significant changes of the basic periods of fluctuations.

Based on the Fast Fourier Transformation, spectral analyses of all simulation runs were carried out. As a result of such analyses, spectral densities are obtained for each run, similar to that presented in Fig. 3 (the run for  $L = 9$ , and  $\lambda = 12$ ). From the analysis of the spectral densities of either the profit/capital ratio or the investment/capital ratio it is possible to identify the basic periods of fluctuations (those with the highest densities). For example, the analysis of the spectral density in Fig. 3 allows us to identify the following basic periods ( $T$ ) of the profit/capital rate: 6.7 years (the highest ordinate), 2.4 years (the second ordinate) and 3.6 years (the 3rd ordinate). The period of 80 years (the first peak on the left side of Fig. 3(a)) is not considered because, in fact, it represents the trend in the 80-element sample. Much-correlated basic periods are for the investment/capital ratio (Fig. 3(b)), but the highest ordinate has a frequency with a period of 2.4 years, the second one is with a period of 6.7 years, and the third one with the period of 3.6 years. It means that the basic periods of the profit and of the investment are the same but the order is slightly different. Very similar pictures are obtained for all other runs.

There is no clear relationship between the computability and the basic periods of fluctuations. A thorough study of fluctuations (based, for example, on the Lyapunov exponents) due to the finite firms' computability is needed, but for our preliminary analysis it is enough to note that the distribution of the basic periods is far from being uniform. The rough cluster analysis of the basic periods observed in all simulation runs shows that there are two clusters: (1) within three to seven years, and (2) about 10 years, and also a few scattered oscillations of longer periods of 16, 20 and 25 years.

In the simulation runs presented in this section it was assumed that computability is constant during simulation and identical for all firms. This assumption allowed a systematic study of the influence of different values of computability on the industry behaviour to be undertaken. Naturally it makes some features of the model's behaviour quite artificial — no two firms have the same computational

ability, and even the same firm is not able to make calculations of the same quality in different periods and for diversified external influences (from other industries, and from other spheres of social life) as well as internal influences (for example, emerging innovations). The unnatural behaviour of the model in the case of constant and uniform value of the firms' computability is clearly visible in some of the figures presented, for example, sharp jumps and very regular, saw-like charts of the profit/capital or the investment/capital ratios. As may be expected, computability is firm specific and embedded in its routines. In general, the firm's computability ought to be described as a stochastic process coupled with the evolution of the firm's routines. Pure random factors may influence the firm's computability, for example, innovation emergence may cause the future industry development to be highly non-deterministic and unpredictable, and the probability of correct expectation to be especially small in the first phase of the innovation diffusion. To get closer to a real situation, in the following series of experiments random changes of either the number of trials  $L$  or the scope of search  $\lambda$  are assumed. This assumption causes a stochastic behaviour of the model. Simulation results of two such experiments are presented in Fig. 4 (pure competition — 12 firms) and in Fig. 5 (oligopoly — four firms). It may be said that the behaviour of the model is a 'sum of

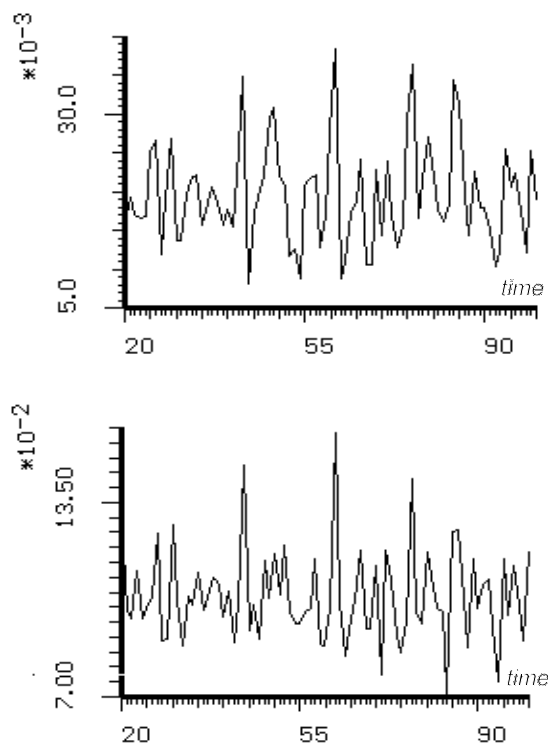


Fig. 4. Profit/capital and investment/capital ratios; diversified firms' computability; pure competition.



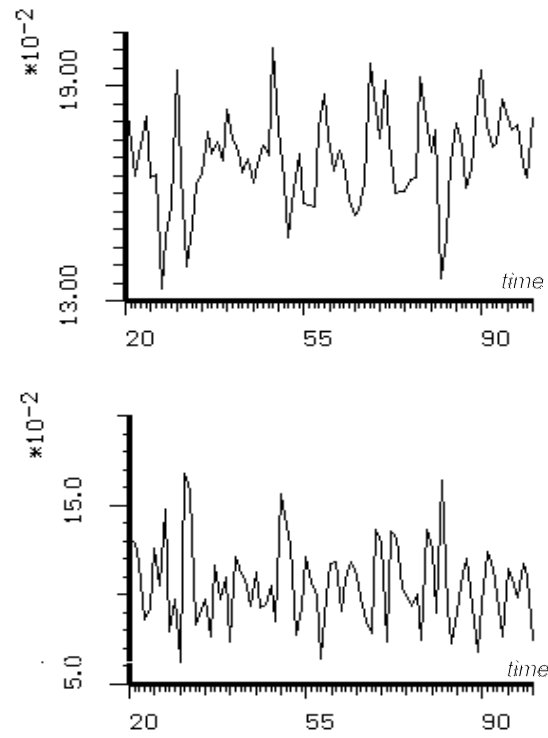


Fig. 5. Profit/capital and investment/capital ratios; diversified firms' computability; oligopoly.

the elementary behaviours' observed in earlier simulation runs. Statistical analysis of numerous simulation runs for diversified firms' computability shows that for such created simulation conditions the emergence of basic fluctuations with periods between three to seven years, and about 10 years can still be observed. In contrast to the former simulation runs where on account of the assumption of constant firms' computability all firms are equal and their market shares do not change during simulations, in this series of simulation runs we observe diversity of firms' size, that is, shares of firms fluctuate around the equilibrium values (8.33% for 12 firms and 25% for four firms). The firms do not choose the same price of products so we also observe relatively high diversity of prices, diversity of investment and diversities of all other firms' characteristics. It may be said that for stochastic firms' computability all industry characteristics fluctuate about their equilibrium values and the industry is in the steady state.

Results presented in this section demonstrate that fluctuations in industry development may occur because of the lack of knowledge of current and future development of the system and because of inaccurate predictions of the behaviour of competitors by economic agents. Naturally there are many other causes of industry fluctuations (for example, 'lags between the initiation of a control action and its effect on the stock, or lags between a change in stock and the perception of that

change by the decision maker'). Analysis of statistical records suggests diversified modes of development of macro-economic systems including short-term business (Kitchin) cycles with periods of about three years, the 9 to 25 years construction (Juglar, Kuznets) cycles, and 45 to 60 years economic long waves, the so-called Kondratieff cycles. Some observations indicate that different modes of economic development interact with one another such that each long wave spans a full number of Kuznets (Juglar) cycles, and each construction cycle a full number of business cycles. Joseph Schumpeter, who was a proponent of such a view [8], opted for three cycles: three Kitchins equalled a Juglar, six Juglars a Kondratieff. In the next section we will see how Kondratieff cycles can emerge in our model due to radical innovations occurrences.

### *Punctuated versus gradual development*

The search for innovation is a result of the interplay of different mechanisms of novelty generation, that is, different strategies of a search. Dichotomously the firms' strategies may be partitioned into: an innovation search (that is, an attempt to search for real novelty through the autonomous, in-house research of a firm) and imitation (that is, a search for innovation through the recombination of some existing solutions). But within the innovation strategy two mechanisms ought to be distinguished: search for novelty through the relatively small modification of current solutions and search for radical novelty through the essential rebuilding (reshaping) of existing solutions. Let us call the innovation strategy through moderate modifications 'mutation' and the search strategy for a radical novelty 'recrudescence'. All these three mechanisms of novelty generation are crucial for long-range economic development, and for all evolutionary processes in general. Mutations enable us to adjust current solutions (technologies) to local environments, to ongoing changes of exogenous conditions, and also to temporal changes of markets' preferences on which the firms operate.

Recombination (imitation) enables relatively quick dissemination (diffusion) of innovations and also enables new solutions to be found through the search for new combinations of existing routines. Collaboration of mutation and imitation enables much quicker development, and provides competitive conditions within the industry, being important forces prohibiting a tendency toward market monopolization. Mutation and imitation act all the time on the same relatively high level, they are vigorous forces allowing each individual firm to keep its position on the market or, with a bit of luck, to reach a temporary superior position. It seems that the practice of the recrudescence is different. As has been said before, the recrudescence reflects phenomena frequently observed in creative processes and described as revelation, vision, bisociation (Arthur Koestler), or gestalt-switch (Karl Popper).

In contrast to imitation and mutation, the recrudescence is hardly detectable during 'normal' research, and may be called a dormant mechanism, but it is highly active during the periods of stagnation, when prospects of current technologies seem

to be exhausted. During these relatively short periods, large numbers of inventions are generated, most of which are useless but some of them open the way for the emergence of radical innovation which focuses the attention of the majority of researchers; in effect the ratio of the recrudescence diminishes. In the succeeding phase of the Kuhnian ‘normal research’, efforts are focused on such promising innovations which are further improved by mutation and recombination. As a hypothesis it may be stated that the ratio of recrudescence is strongly correlated to the economic state of affairs — during periods of prosperity the recrudescence is almost invisible but emerges and gains vital status during relatively short periods of depression and stagnation. In reality all mechanisms of novelty generation act concurrently. It seems interesting to isolate each mechanism and study the impact of each separated mechanism on the modes of industrial development. Adaptive landscapes describing the performance index (technical competitiveness) are defined in the space of technical characteristics —  $q(z)$  in Eq. (14). As may be expected real adaptive landscapes are dynamic entities with many local peaks. The adaptive landscape’s surface depends on the evolution of the industry under consideration as well as on the co-evolution of other related industries, but also, in general, on the whole socio-economic evolution. In principle it is possible to model such a complicated landscape by relevant definition of function  $q(z)$ , but to control the results of experiments it is better to start the simulation with simple, stable adaptive landscapes. In the following experiment it is assumed that there are only two technical characteristics, the adaptive landscape does not change its shape during the simulation and there are four local peaks with altitudes equal to 1.0, 1.5, 2.0 and 2.5. Values of  $q(z)$  reflect relative preferences of different solutions. Multiplication of  $q(z)$  by any positive number does not change the shape of the landscape and the behaviour of the model. It means that solutions around the second higher peak provide 50% better performance than the solutions around the lowest peak. The map of this adaptive landscape is presented in Fig. 6. The initial values of the product characteristics are much closer to the lowest peak

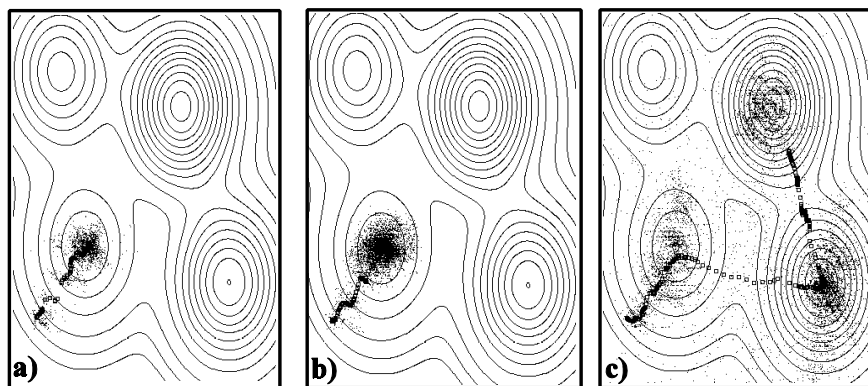


Fig. 6. Trajectories of development: (a) mutation, (b) recombination and (c) recrudescence.

so we may expect that the trajectory of evolution at the first stage of the industry development will evolve toward that peak and then that the firms will try to find better products with characteristics closer to higher peaks. It is important, and ought to be emphasized, that the firms do not know the shape of the adaptive landscape and the only way to gain knowledge about the local shape of the landscape is to make an experiment — during the R&D process firms evaluate the performance index, that is, the technical competitiveness, of a specific product with assumed values of characteristics. All such experiments made by all firms during the whole period of simulation are marked by dots (pixels) on the background of the adaptive landscape in Fig. 6. The performance index (that is, technical competitiveness) of products defined by known values of their characteristics marked by dots is known for firms (and only this part of the adaptive landscape is known for individual firms, that is, those firms which make a specific ‘experiment’). It may be said that dots mark all inventions found by the firms as the result of R&D process. The number and density of the dots in all three charts in Fig. 6 also suggest differences in the vigorousness of the search process. Some of the inventions are adopted by firms and become innovations, that is, products offered for sale on the market. Average values of characteristics of products sold on the market at any time  $t$  are marked by squares. We say that the average values of product characteristics sold on the market mark the trajectory of industry development in the adaptive landscape.

In the first experiment it was assumed that only mutation acts. The development of each firm is based only on its own knowledge and on autonomous research. The firms evolve almost directly through the shortest way toward the lowest peak. The scope of search for invention is not very large (Fig. 6(a)), and the research is focused around local firms’ positions in the adaptive landscape. If we add the possibility of interchanging knowledge (that is, imitation of innovation) the evolution is slightly quicker (but the routines diversity is much smaller). The scope of search is also slightly wider than in the former experiment. Let us note that the trajectories of development in these two experiments significantly differ; the simulation conditions, besides the modes of research, in these experiments are exactly the same. Maximum and average technical competitiveness in these two experiments are presented in Figs. 7(a) and 7(b), respectively. It is seen that imitation (recombination) allows for more smooth development; the discrepancy between the frontier of development (represented by the maximum technical competitiveness) and average values are much smaller when imitation is allowed.

In both cases the evolution stops at the lowest peak. Greater values of probabilities of mutation and recombination accelerate evolution and lead to a relatively high ratio of technological development but still do not allow for departure from the lower local peak (local optimum, as it is sometimes called) through finding products with characteristics very close to the higher peaks. We use the term ‘evolutionary trap’ to name the situation of confining the industry in the local, lower peak of the adaptive landscape. Many other simulation runs with different adaptive landscapes let us conclude that neither mutation nor recombination (imitation) allow us to

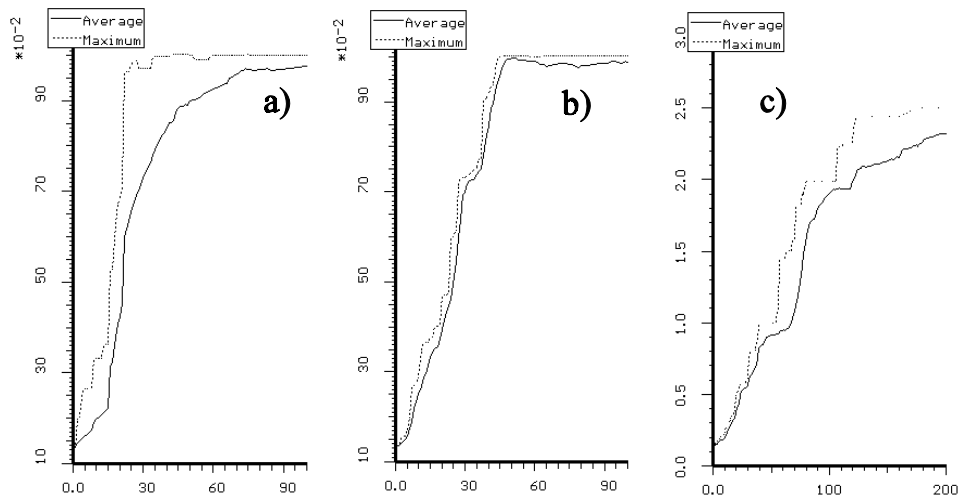


Fig. 7. Quality of performance (technical competitiveness) for (a) mutation, (b) recombination and (c) recrudescence.

escape from the majority of evolutionary traps. As our simulation experiments reveal, the mechanism of recrudescence makes this escape much easier. In the next simulation experiment this mechanism is added. In the first period (up to 50 years) mutation and imitation act at the normal levels, as in the former experiment, and recrudescence acts rarely. The industry development is similar to that in the previous runs. At  $t = 50$  industry is very close to the first lowest peak and at this moment we allow the recrudescence to act on a much higher level; within 5 years products with characteristics very close to the higher peak (with altitude equal to 2.0) are found. At  $t = 55$  the probability of recrudescence is reduced to the lower value. The scope of search in this run is much wider than in all previous runs, Fig. 6(c). Far-distanced areas are sampled but most of these attempts are fruitless. Not all far-placed inventions are generated by recrudescence; most of them are the result of a recombination of solutions placed at different peaks, but the first inventions placed at the higher peak are always generated by recrudescence and open the way for the recombination of products 'placed' at these two peaks. After reaching the third peak, recrudescence with higher probabilities is allowed again and the solutions on the highest peak are found (see the trajectory of development in Fig. 6(c), technical competitiveness in Fig. 7(c)).

It may be said that recrudescence acts as a trigger, initiating the phase of radical transformations. Not all inventions providing better products performance are accepted; frequently modifications of routines which generate technical inventions placed at the higher peak also cause reduction of productivity of capital or a rise in the unit costs, and therefore they are not accepted simply on the basis of economic judgements. The necessity of correlation of technical performance with economic factors (as productivity of capital and costs of production, but also other

factors, for example, a firm's current investment capabilities) causes many promising inventions to be rejected by firms, and in practice the probability of the emergence of radical innovation is significantly smaller than the probability of finding radical technical invention.

The emergence of radical innovation is a kind of leap, a punctuated process, but the shift from the lower to the higher peak is not a sharp (punctuated) process; rather, it is a much more gradual process of shifting the position of the industry in the adaptive landscape. The main reason for this gradualism is that the overall competitiveness of products is the function of the technical competitiveness and the price — see Eq. (4). To keep the overall competitiveness on a relatively high level, firms lower the price of products characterized by smaller technical competitiveness (that is, placed at the lower peak) and vice versa products with higher competitiveness (that is, placed at the higher peak) are slightly more expensive (to gain greater profit), so the values of the overall competitiveness for the products of firms in the vanguard of technological development are only slightly greater than the competitiveness of the old-fashioned products. Therefore the elimination of the worst products from the market is not so sharp as may be expected on the basis of the values of technical competitiveness only. In some circumstances the substitution phase may last quite a long time, but in all cases we observe the steady tendency to reduce production of the old-fashioned products and to increase the production of the modern ones.

If a recrudescence mechanism is involved, jumps in the development of the technological frontier are clearly visible — see the maximum technical competitiveness in the Fig. 7(c). The jumps are observed on the route toward the local peak and also in the transition phase, of passing from the lower to the higher peaks.

Random factors play a crucial role in the evolution of the industry. We may say that the route toward local optimum is more or less predetermined; but after reaching the local optimum further development in multi-peak landscape is hardly predictable. The highest peak can be reached by different routes. In our example it may happen that the best solutions are found just after reaching the lowest peak (see Fig. 8(b)) or indirectly through the second lower peak as in Fig. 8(a).

Specific simulation run is presented in Fig. 8(c). As we see, the trajectory goes somewhere between the highest and the second highest peaks. It is because some firms are placed on the highest peak and some other on the third peak. What is interesting, this partition exists in spite of essential differences in the technical competitiveness. This specific situation is not rare one in economic processes. We mention it because this case reflects some specificities of economic evolution. As it was mentioned, the overall competitiveness depends on technical competitiveness and price (see Eq. (14)). It happens that higher technical competitiveness of products of firms placed on the highest peak is accompanied by greater unit cost of production and vice versa, smaller technical competitiveness of products of firms placed on the lower peak is accompanied by lower cost of production. Therefore

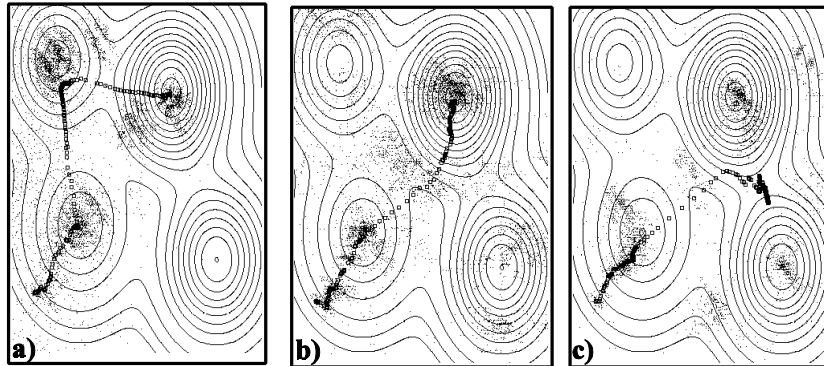


Fig. 8. Trajectories of development (recrudescence, different runs).

firms on the highest peak are forced to charge higher price for their production and firms on the lower peak charge lower price (because smaller unit cost). In consequence the overall competitiveness of both kind of firms is almost equal. Some consumers decide to buy better products for higher price some other consumers buy worse products for smaller price. The market share of firms placed on the highest peak and the share of firms placed on the lower peak are almost the same (in some simulations even the share of firms producing less technologically advanced products increase).

Figure 9 summarizes all results presented in Figs. 6 and 8. It is visible that trajectories of industry development are more or less gradual when firms evolve toward a local peak along the hill and is punctuated when after a relatively short period of being 'trapped' on a local peak a radical innovation, placed on the higher peak, is found. We have used the same landscape with four peaks in simulations of biological populations. Price mechanisms and pre-evaluation of new solutions (inventions) before 'launching' them into the environment (market) are distinguished features of industrial development from biological evolution. The differences between economic and biological evolutions make that for some simulation conditions, modes of development can be different in both systems. The trajectories in biological evolution look very similar to that presented in Fig. 9 (with phased of gradual and punctuated modes of development) for large biological populations. Large populations are also not able to escape from the evolutionary trap in gradual way. Gradual process of development dominates in small biological populations (in our simulations, less than 15 individuals). In small biological populations it is possible to detect presence of genetical drift. Random changes of heritage information of the small population (genetic pool) cause that the population is able to evolve down the local peak toward a valley between lower and higher peaks and in the next phase the population is able to evolve gradually along the hill of the higher peak to reach the top of that hill.

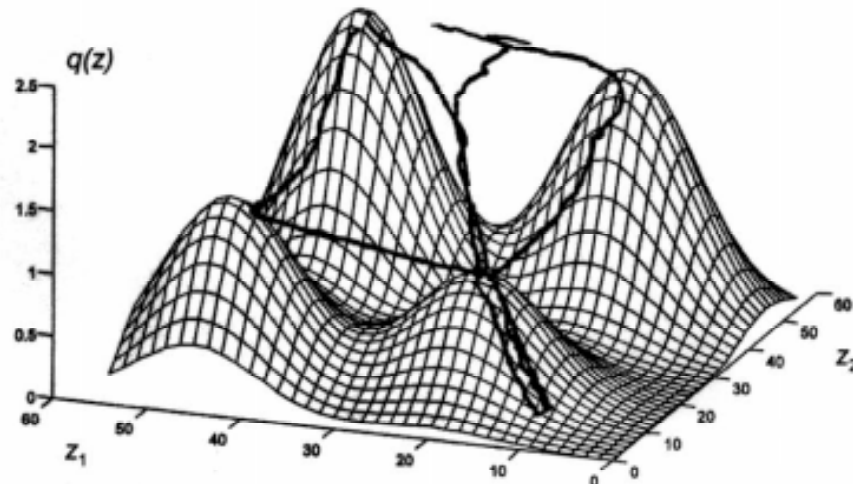


Fig. 9. “Look! Here he comes, leaping across the mountains, bounding over the hills” (Song of Solomon 2:8).

## Conclusions

Evolutionary processes are dynamical, historical ones where macro-characteristics flow from activities of individual agents. Fundamental features of evolutionary processes are diversity and heterogeneity of behaviours. Selection and search for innovation are two basic mechanisms of development. But we ought to be conscious that each particular evolutionary process has its own peculiarities, such that one related to investment, capital formation and price setting in economic evolution. Mechanisms of search for innovation seem to be the common property of all evolutionary processes, and in fact this part of the industrial model is ‘borrowed’ from our former model of biological evolution. It is reflected also in the nomenclature used, such as mutation, recombination, and so on, so well known in biological models.

Fluctuation of 3–6 years and of 10 years periodicity can occur in an industry development because of firms’ bounded rationality. Long waves of development of 50–60 years period (Kondratieff cycles) occur in our model because of radical innovation emergence at the maturity phase of an ‘old’ technology.

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