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# Market, innovation, competition
## An evolutionary model of industrial dynamics

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A model is presented of the behavior of a number of competing firms producing functionally similar products. Each firm searches for new routines and new combinations of the routines, i.e. 'technical and organizational ideas and skills associated with a particular economic context'. Decisions of a firm related to investment, modernization of production, price, etc. are based on the firm's evaluation of the behavior of other competing firms and the expected response of the market. Firms search for new combinations of routines in order to minimize the unit cost of production, maximize the productivity of capital, and maximize the competitiveness of their products in the market. Results of simulation of the model concerning investigation of price setting procedures, and long- and short-term firms' objectives are presented.

## 1. Introduction

This paper presents a model that describes the behavior of a number of competing firms producing functionally equivalent products. Each firm searches for new routines and new combinations of routines. Nelson and Winter (1982, p. 14) define routines as 'regular and predictable behavioral patterns of firms' and include in this term such characteristics of firms as 'technical routines for producing things, (...) procedures of hiring and firing, ordering new inventory, stepping up production of items in high demand, policies regarding investment, research and development, advertising, business strategies about product diversification and overseas investment.'

The decisions of a firm relating to investment, modernization of production, price, etc. are based on the firm's evaluation of behavior of other competing firms and the expected response of the market. The firm's knowledge of the market and knowledge of the future behavior of competitors is limited and uncertain. There is no possibility of characterizing the

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limitation and the uncertainty of knowledge in statistical terms, e.g., in terms of probability distributions. Firms' decisions can only be suboptimal.

Each firm tends to improve its situation within the industry and in the market by introducing new combinations of routines in order to minimize the unit cost of production, maximize the productivity of capital, and maximize the competitiveness of its products in the market. Productivity of capital, unit cost of production, and characteristics of products manufactured by a firm depend on the routines employed by the firm (examples of the characteristics are: Reliability, convenience, lifetime, safety of use, cost of use, quality, aesthetic values). The search activity of firms 'involve the manipulation and recombination of the actual technological and organizational ideas and skills associated with a particular economic context' [Winter (1984)], while the market decisions depend on the products' characteristics and prices. We may speak about the existence of two spaces: The space of routines and the space of product characteristics.[1] Distinguishing these two spaces enables us to separate firms' decisions from the markets' decisions. We assume discrete time in the model, e.g., a year or a quarter. The firms' decisions relating to investment, production, research funds, etc. are taken simultaneously and independently by all firms at the beginning of each period. After the decisions are made the firms undertake production and put the products on the market. The products are evaluated by the market, and the quantities of different firms' products sold in the market depend on the relative prices, the relative value of products' characteristics and the level of saturation of the market. Frequently, as the result of collective phenomena, the products evaluated as the best are not sold in the full quantity offered, and conversely, the inferior products are frequently sold in spite of the possibility of buying the better ones. But during long periods the preference for better products, i.e. those with a lower price and better characteristics, prevails.

In the model presented below each firm may simultaneously produce products with different prices and different values of the characteristics, that is, the firm may be a multi-unit operation. Different units of the same firm manufacture products by employing different sets of routines. Multi-unit firms exist because of the searching activity. New technical or organizational

[1]A space of routines and a space of characteristics play in our model an analogous role to a space of genotypes and a space of phenotypes in biology. The existence of these two types of spaces is a general property of evolutionary processes [Kwasnicka and Kwasnicki (1986)]. Probably the search spaces (i.e., spaces of routines and space of genotypes) are discrete spaces contrary to the evaluation spaces (i.e., space of characteristics and space of phenotypes) which are continuous spaces. The dimension of the space of routines (space of genotypes) is much greater than the dimension of the space of characteristics (space of phenotypes). As some simulation experiments reveal, big differences in the dimensions of the two spaces play an important role in long-term evolution and among others enables escape from so called evolutionary traps.

solutions (i.e., new set of routines) may be much better that the actual ones but full modernization of production is not possible because of investment constraints on the firm. In such situations the firm continues production employing the old routines and tries to open a new unit where production, on a lesser scale, employing the new set of routines is started. Subsequently the 'old' production may be reduced and after some time superseded by the 'new' production.

## 2. The model

Simulation of industry development in the model is made in discrete time in four steps:

(1) Search for the new sets of routines which potentially may replace the 'old' set currently employed by a firm.
(2) Calculation and comparison of the investment, the production, the net income, the profit, and some other characteristics of development which may be attained by employing the 'old' and the 'new' sets of routines. Decisions of each firm on: (a) continuation of production by employing old routines or making modernization of production; and (b) opening (or not) of new units.
(3) Entry of new firms.
(4) Market evaluation of the offered pool of products. Calculation of firms' characteristics: Production sold, shares in global production and global sales, total profits, profit rates, research funds, etc.

### *2.1. Search process*

We assume that at time $t$ a firm unit is characterized by a set of routines actually employed by the firm. There are two types of routines: *Active*, i.e., routines employed by this firm in its everyday practice, and *latent*, i.e., routines which are stored by a firm but not actually applied. Latent routines may be included in the active set of routines at a future time. The set of routines is divided into separate subsets, called segments, consisting of similar routines employed by the firm in different domains of the firm's activity. Examples are segments relating to productive activity, managerial and organizational activity, marketing, etc. In each segment, either active or latent routines may exist.

The set of routines employed by a firm may evolve. There are four basic mechanisms of generation of new sets of routines, namely: *Mutation, recombination, transition* and *transposition.*

The probability of discovery of a new routine (*mutation*) depends on the research funds allocated by the firm for autonomous research. The firm may also allocate some funds for gaining knowledge of other competing firms and try to imitate (*recombination*) some routines employed by competitors. It is assumed that recombination may occur only between segments, not between individual routines, i.e., a firm may gain knowledge about whole domain of activity of another firm e.g., by licensing. A single routine may be transmitted (*transition*) with some probability from firm to firm. It is assumed that after transition a routine belongs to a subset of latent routines. At any time a random *transposition* of a latent routine to a subset of active routines may occur. A more detailed description of the four basic mechanisms of evolution of routines is presented in the following sections.

#### 2.1.1. *Research funds.*

It is assumed that R&D fund ($R_i$) allocated by a firm into research (*innovation* and *imitation*) are a function of actual firms capital ($K_i$) of the firm.

$$R_i = (h_2 \exp(-h_1 K_i) + h_0) K_i. \tag{1}$$

Research funds are proportional to a firm's capital if $h_1$ and $h_2$ are equal to zero. If $h_1$ and $h_2$ are greater than zero small firms allocate a greater percentage of their capital into research and a local maximum of R&D funds will appear near $K_i = 1/h_1$. Total R&D funds are partitioned into funds ($\mathrm{Rm}_i$) for innovation (*mutation*) and funds ($\mathrm{Rr}_i$) for imitation (*recombination*). The strategy of research of firm $i$ at year $t$ is described by the coefficient ($g_i$) of partition of total R&D expenditure into innovation and imitation.

$$\mathrm{Rm}_i = g_i R_i \qquad \mathrm{Rr}_i = (1 - g_i) R_i. \tag{2}$$

The strategy of research changes from year to year and depends on the actual state of affairs of a firm. It is assumed that the share of research on innovation increases if the firm's share in global production is increasing (i.e., if assumed position of the firm on a background of other competing firm is good). If a firm's share decreases, more funds are allocated in imitation, i.e., a firm supposes that there are other firms applying better technology and it is better and safer to search for these technologies. The rate of change of coefficient $g_i$ depends on the size of a firm and it is smaller the larger the firm is.

$$g_i(t+1) = (1 + (G/K_i)(f_i(t) - f_i(t-1))/f_i(t-1)) g_i(t), \tag{3}$$

where $g_i(t)$ is the coefficient of R&D funds partition at time $t$, $G$ is the constant controlling rate of change of $g_i$, and $f_i(t)$ is the share of firm $i$ in global production at time $t$.

During any year of searching activity more than one set of new routines $r'$

may be found. The number of such alternative sets of routines, the so called number of experiments, is a function of research funds.

$$\mathrm{NoExp}_i = \mathrm{round}(e(R_i)^{\psi}) + E_0, \tag{4}$$

where NoExp is the number of experiments of firm $i$, $e$, $\psi$, $E$ are coefficients with the same values for all firms, and $R_i$ is the R&D expenditure of firm $i$.

*2.1.2. Mutation.*

We assume that routines mutate independently of each other. Since the range of the routines is bounded, we numerate all possible routines and assume that the range is from MinRut to MaxRut.

Let $r_{lk}$ denote the $l$th routine in the $k$th segment employed by a firm in period $(t-1, t)$. After mutation routine $r_{lk}$:

(1) is not changed, i.e., $r'_{lk} = r_{lk}$, with probability $(1 - \mathrm{PrMut})$, or
(2) is changed and is equal to

$$r'_{lk} = r_{lk} + x, \quad x \in (-\mathrm{MaxMut}, \mathrm{MaxMut}),$$

with probability PrMut/(2MaxMut) for every $x$.

The probability of mutation of a routine depends on R&D funds allocated by firm $i$ to search for innovations.

$$\mathrm{PrMut}_i = a^{\mathrm{m}}(\mathrm{Rm}_i)^{\zeta} + b^{\mathrm{m}}, \tag{5}$$

where $a^{\mathrm{m}}$, $\zeta$ are coefficients controlling probability of mutation, and $b^{\mathrm{m}}$ is the probability of mutation related to the public knowledge.

*2.1.3. Recombination*

A firm $i$ may get knowledge about the routines of a single segment of a firm $j$ with probability PrRec. At the same time the firm $i$ may get knowledge employed by different firms, so new sets of routines may consist of routines of different firms. In the model the firm $i$ may apply one of three strategies of recombination:

(1) Conditional probability of recombination of segment $k$ of firm-unit $i$ with segment $k$ of firm-unit $j$ is proportional to the share of firm-unit $j$ in global production.
(2) Conditional probability of recombination of segment $k$ of firm-unit $i$ with segment $k$ of firm-unit $j$ is proportional to the rate of expansion of firm-unit $j$, i.e., is proportional to the derivative of the share of firm-unit $j$.
(3) Conditional probability of recombination of segment $k$ of firm-unit $i$

with segment $k$ of firm-unit $j$ is reciprocal to the number of firms existing in the market, i.e., is equal for each firm-unit $j$.

The probability of recombination of a segment is a function of R&D funds allocated to imitation.

$$\mathrm{PeRec}_i = a^{\mathrm{r}}(\mathrm{Rr}_i)^{\xi} + b^{\mathrm{r}}, \tag{6}$$

$a^{\mathrm{r}}$, $\xi$ are coefficients controlling probability of recombination, and $b^{\mathrm{r}}$ is the probability of recombination related to the public knowledge.

### 2.1.4. Transition, transposition and recrudescence

We assume that the probabilities of transition of a routine from one firm to another and the probabilities of transposition of a routine (from a latent to an active routine) are independent of R&D funds, and have the same constant value for all routines. In general, the probability of transposition of a routine for any firm is rather small. But randomly, from time to time, the value of this probability may abruptly increase and we observe very active processes of search for new combination of routines. We call this phenomena recrudescence.[2]

We view recrudescence as an intrinsic ability of a firm's research staff to innovate by employing some reckless, insane ideas. This ability is connected mainly with the personalities of the researchers. Pure random factors play an essential role in searching for innovations by recrudescence, so the probability of recrudescence is not related to R&D funds allocated by a firm to 'normal' research.

We assume that recrudescence is more probable in small firms than in large ones which spend huge quantities on R&D. The probability of recrudescence in firm $i$ is equal to

$$\mathrm{PrRence}_i = u_1 \exp(-u_2 K_i). \tag{7}$$

As a rule mutation, recombination and transposition on a normal level (i.e., with low probabilities in long periods) are responsible for small

[2]Recrudescence comes from Latin *recrudesco* (to break out, to open, to renew) and *recrudescere* (to becomed raw again). Recrudescence means a new outbreak after a period of abatement, inactivity or after a dormant period. We suggest that the mechanism of generation of innovation by recrudescence is a general mechanism observed in all evolutionary processes, e.g., in biological evolution, in development of knowledge, and in economic development. Results of simulations of models of biological evolution and the model of industry development suggest that recrudescence is essential for long-term evolution and permits to escape from evolutionary traps. In economies recrudescence is mainly responsible for emergence of new technological regimes (Nelson and Winter), new technology paradigms (Dosi) and new technological systems (Freeman, Clark and Soete).

improvements and in short periods of recrudescence for the emergence of radical innovations.

### 2.2. Firms' decisions

Productivity of capital, variable cost of production and product characteristics are functions of the routines employed. Each routine has multiple pleiotropic effects, i.e., may affect many products characteristics, as well as productivity, and the variable cost of production.

We assume that the transformation of the set of routines into the set of products' characteristics is described by $m$ functions $F_d$,

$$z_d = F_d(r), \quad d = 1, \ldots, m, \tag{8}$$

where $z_d$ is the value of $d$ characteristic, $m$ the number of characteristics, and $r$ the set of routines.

A product's attractiveness on the market depends on values of the product's characteristics and its price. We assume the existence of a function $q$ enabling calculation of competitiveness [term proposed by Silverberg (1987)] of products manufactured by different firms.

Competitiveness of products with characteristics $z$ and price $p$ is equal to

$$c(p, z) = q(z)/p^{\alpha}, \quad z = (z_1, z_2, z_3, \ldots, z_m), \tag{9}$$

where $q(z)$ is the technical competitiveness, $z$ a vector of products' characteristics, and $\alpha$ the elasticity of price in the competitiveness.

Denote by $c(p, z)$ the competitiveness of products actually manufactures by firm-unit $i$, i.e., the firm-unit employs the set of routines $r$, and by $c(p, z')$ the competitiveness of products after modernization, i.e., the firm-unit employs the new set of routines $r'$. The decision of firm $i$ about making modernization depends on the expected profits and investment capabilities of the firm. In both cases, $r$ and $r'$, the firm estimates expected maximal profit which may be attained in the current state of the market. Modernization is made if the profit in case $r'$ is greater than the profit in case $r$ and if the investment capability of the firm permits such modernization. If the investment capability does not allow the modernization, then the firm continues production employing the old routines $r$ and tries to open a new small unit where production on a lesser scale employing routines $r'$ is started.

The procedure of evaluation of production, investment, and expected income and profit in the next period of time of firm-unit $i$ employing routines $r^{\mathrm{r}}$ ($r^{\mathrm{r}} = r$ or $r^{\mathrm{r}} = r'$) at product price $p_i(t)$ is presented below:

(a) Calculation of the competitiveness $c_i(t) = c(p_i(t), z^{\mathrm{r}})$.

(b) Estimation of the average price and average competitiveness. The firm assumes that in the next period competitors will behave in a similar way as in the past and that in the period $(t, t+1)$ the average price will be equal to

$$p^{e}(t) = p^{p}(t)(1 - f_i(t-1)) + p_i(t) f_i(t-1). \tag{10}$$

Similarly, the average competitiveness is assumed to be equal to

$$c^{e}(t) = c^{p}(t)(1 - f_i(t-1)) + c_i(t) f_i(t-1), \tag{11}$$

where $f_i(t-1)$ is the market share of firm-unit $i$ in the previous period of time, and $p^{p}(t)$ and $c^{p}(t)$ are predicted values of average price and average competitiveness, respectively. They are equal to

$$p^{p}(t) = p^{e}(t-1)(p^{e}(t-1)/p^{e}(t-2))^{\tau}, \tag{12}$$

$$c^{p}(t) = c^{e}(t-1)(c^{e}(t-1)/c^{e}(t-2))^{\tau}. \tag{13}$$

(c) Estimation of the global production. We assume that all firms know the demand function. The quantity of products potentially sold on a market is equal to an amount of money – $M(t)$ – which the market is inclined to spend on buying products offered by the firms divided by the average price – $p^{e}(t)$ – of products offered by these firms, i.e., the demand function is equal to

$$Q(t) = M(t)/p^{e}(t). \tag{14}$$

We assume that

$$M(t) = N \exp(\gamma t)(p^{e}(t))^{\beta}, \tag{15}$$

where $N$ is a parameter characterizing the initial market size, $\gamma$ the growth rate of the market size, and $\beta$ the elasticity of the average price.

(d) Estimation of the market share of firm-unit $i$. The share of firm-unit $i$ in period $(t, t+1)$ is equal to

$$f_i(t) = f_i(t-1)(c_i(t)/c^{e}(t)). \tag{16}$$

(e) Estimation of the production of firm-unit $i$ (to be accepted by the market).

$$Q_i(t) = f_i(t) Q(t). \tag{17}$$

Capital needed to produce output $Q_i(t)$ is equal to

$$K_i(t) = Q_i(t)/A(r^r), \tag{18}$$

where $A(r^r)$ is the productivity of capital by employing the set of routines $r^r$.

If required growth of the capital of firm-unit $i$ is greater than the investment capability of firm $i$ then it is assumed that the capital of firm-unit $i$ at time $t$ is equal to the sum of the investment capability and the capital at $t-1$, minus the capital physical depreciation. For such calculated capital the production $Q_i(t)$ is recalculated as

$$Q_i(t) = K_i(t)A(r^r). \tag{19}$$

(f) Estimation of the expected income and profit. The expected income and profit of firm-unit $i$ are equal to

$$\Gamma_i = Q_i(t)(p_i(t) - V(r^r)v(Q_i(t)) - \eta), \tag{20}$$

$$\Pi_i = \Gamma_i - K_i(t)(\rho + \delta) - R_i(t), \tag{21}$$

where $\Gamma_i$ is the expected income of firm-unit $i$ at time $t+1$, $\Pi_i$ is the expected profit of firm-unit $i$ at time $t+1$, $Q_i(t)$ the output of firm-unit $i$, $V(r^r)$ the variable production cost as a function of employed routines $r^r$, and $v(Q_i)$ a factor of unit production cost as the function of a scale of production, in the model assumed to be equal to

$$v(Q) = \exp(-\varepsilon Q), \quad \varepsilon > 0,$$

$\eta$ is the constant production cost, $K_i(t)$ the capital needed to manufacture the output $Q_i(t)$, $\rho$ the normal rate of return, $\delta$ the physical capital depreciation rate, and $R_i(t)$ denotes the R&D expenditure. We assume that the productivity function $A(r)$ and the cost functions $V(r)$ and $v(Q)$ are common for all firms.

To modernize production it is necessary to incur an extra investment. The modernization investment depends on the discrepancy between the old routines $r$ and the new routines $r'$. For simplicity of calculation, we asume that the modernization investment is a function of technical competitiveness, productivity of capital and variable cost of production, and is equal to

$$\begin{aligned}\mathrm{IM}_i(t) &= K_i(t)F(q(z), q(z'), k_1) \\ &\quad \times F(A(r), A(r'), k_2)F(V(r), V(r'), k_3), \end{aligned} \tag{22}$$

where $F$ is the function defined as

$$F(x,y,z) = (\max\{x/y, y/x\})^z - 1,$$

$k_1$, $k_2$, $k_3$ are coefficients controlling impact of the technical competitiveness, the productivity of capital, and the variable cost of production on the modernization investment.

The investment capability of firm $i$ in period $(t, t+1)$ is a function of profits of all units of the firm $i$ in period $(t-1, t)$,

$$\mathrm{IC}_i(t) = \delta\left(\sum_l K_l(t-1)\right) + \mu\left(\sum_l \Pi_l(t-1)\right), \tag{23}$$

where $\delta$ is the physical capital depreciation, and $\mu$ a coefficient equal to 1 for $\sum_l \Pi_l < 0$, and equal to $\mu_0$ for $\sum_l \Pi_l > 0$, (the sum is for all units $l$ if the firm $i$). The credit parameter $\mu_0$ is greater or equal to one.

The firm $i$ takes credit if its overall investment $I_i(t)$ at time $t$ (i.e., the sum of the expansionary investment, the modernization investment, and the investment required to open new units) exceeds the sum of the amortization and the profit of the firm at $(t-1)$. The interest rate of the credit is equal to the normal rate of return $\rho$, the average period of repayment is equal to $\mu_1$ years. The debt of firm $i$ is repaid from the profit at time $t$.

The debt of the firm $i$ at time $t$ is equal to

$$D_i(t) = D_i(t-1)(1 + \rho - 1/\mu_1) + \max\{I_i(t) - \delta K_i(t-1) - \Pi_i(t-1), 0\}. \tag{24}$$

The firm makes modernization if the expected profit after modernization is greater than the expected profit in the case of continuation of production by employing the old routines $r$ and if the investment capability enables such modernization. If the modernization is not possible then the firm:

(1) continues production by employing the old routines;
(2) tries to open a new unit where routines $r'$ are employed.

Production is started with an assumed value of the capital, *InitCapital*. The firm calculates expected profits for two cases: First, if in the new unit new routines $r'$ are employed, and second, if old routines $r$ would be employed in this unit. The new unit is opened if the profit in the first case is greater than in the second one.

### 2.3. Entry of new firms

In each period $(t, t+1)$ a number of firms try to enter the market. The probabilities of mutation and recombination for the potential entrants are assumed and the set of routines for each entrant is generated on the base of the pool of routines of existing firms. The firms try to enter the market with capital equal to *InitCapital* and with the initial price of their products equal to $(q(z)/c^{p}(t))^{1/\alpha}$, i.e., the initial price makes the competitiveness of the entrants equal to the predicted average competitiveness.

As the result of competition the market shares of firms with competitiveness smaller than average competitiveness decrease, and the shares of firms with competitiveness greater than average competitiveness increase. A firm is driven from the market if it does not keep pace with competitors (i.e., in the long run its competitiveness is smaller than the average competitiveness). To limit the number of very small firms we assume that a firm is eliminated from the register of firms if its market share is smaller than some assumed minimum share, e.g., 0.1%.

### 2.4. *Competition of products in the market*

All products manufactured by the entrants and the firms existing in the previous period are put on the market and evaluated.

We assume that the quantity of products – $Q(t)$ – potentially sold on a market is equal to an amount of money – $M(t)$ – which the market is inclined to spend on buying products offered by the firms divided by the average price – $p^{e}(t)$ – of products offered by these firms [see eqs. (14) and (15)].

The average price of products is equal to

$$p^{e}(t)=\sum_{i} p_i(t)(Q_i(t)/Q'(t)). \tag{25}$$

Total output offered for sale is equal to

$$Q'(t)=\sum_{i} Q_i(t). \tag{26}$$

Global production sold is equal to

$$QS(t)=\min\{Q(t),Q'(t)\}. \tag{27}$$

The general equations of firm competition in a market has the following form,

$$f_i(t)=f_i(t-1)c_i(t)/c^{e}(t), \tag{28}$$

where $c^{e}(t)$ is the average competitiveness of products offered for sale,

$$c^{e}(t)=\sum_{i} f_i(t-1)\,c_i(t). \tag{29}$$

This means that the share $f_i$ of firm-unit $i$ in global output increases if the competitiveness of its products is greater than the average competitiveness of all profits offered for sale, and decreases if the competitiveness is less than the average competitiveness. The rate of change is proportional to the differences between the competitiveness of firm $i$'s products and average competitiveness.

The quantity of products sold by the firm on the market is equal to

$$\mathrm{QS}_i(t)=\mathrm{QS}(t)\,f_i(t). \tag{30}$$

The above equations are valid if the production offered by firms fits exactly the demand of the market. This is a very rare state and therefore these equations have to be adjusted to states of discrepancy between global demand and global production, and of discrepancy between the demand for products of specific firm and the production offered by this firm.

The equations below describe such adjustment. The general meaning parallels that of eqs. (28)–(30). If supply exactly meets market demand (i.e., if $Q(t)=Q'(t)$ and $Q_i(t)=Q_i'(t)$ for all $i$), eqs. (31)–(35) are equivalent to eqs. (28)–(30).

Production sold by firm-unit $i$ is equal to

$$\begin{aligned}\mathrm{QS}_i(t)=&\min\{Q_i(t),Q_i'(t)\}\\&+Q''(t)\max\{0,Q_i(t)-Q_i'(t)\}\,c_i(t)/c''(t),\end{aligned} \tag{31}$$

$$Q_i'(t)=wQS(t)\,f_i(t-1)\,c_i(t)/c^{e}(t), \tag{32}$$

$$w=\min\{1,Q(t)/Q'(t)\}, \tag{33}$$

$$Q''(t)=\sum_{i}\max\{0,Q_i(t)-Q_i'(t)\}, \tag{34}$$

$$c''(t)=\sum_{i}\max\{0,Q_i(t)-Q_i'(t)\}\,c_i(t), \tag{35}$$

where $Q''$ is the sum of production of all firms which produce more than the specific demand for their products, and $c''$ is the average competitiveness of this part of production.

The coefficient $w$ divides the behavior of the model into two regimes: $w$ is equal to one if the demand exceeds the supply, and is smaller than one in another case. If there is no global oversupply (i.e. $w=1$) then the products of the firms which produce more than the demand are sold instead of the potential production of the firms which produce less than the demand [second expression in eq. (31)]. If there is global oversupply then $w \cdot 100\%$ of the demand is supplied by the production of each firm [first expression in eq. (31)]. The rest $(1-w)100\%$ of the demand is supplied from the $Q''$ production, but the share of firm $i$ in the $(1-w)100\%$ part of the production is proportional to the competitiveness of products of this firm [2nd expression in eq. (31)].

The market share of the production sold of firm-unit $i$ is equal to

$$f_i(t) = \mathrm{QS}_i(t)/\mathrm{QS}(t). \tag{36}$$

The real profit of firm-unit $i$ is equal to

$$\begin{aligned} \Pi_i = {} & Q_i(t)[p_i(t) - V(r^{\mathrm{r}})v(Q_i(t)) - \eta] \\ & - K_i(t)(\rho+\delta) - R_i(t) - D_i(t)/\mu_1, \end{aligned} \tag{37}$$

$$\Pi_i = \Gamma_i - K_i(t)(\rho+\delta) - R_i(t) - D_i(t)/\mu_1, \tag{38}$$

where $r$ is the set of routines employed by firm-unit $i$. $K_i(t)$ in eqs. (37), (38) is the value of capital allocated by firm $i$ to produce the output $Q_i(t)$, so profits are smaller than expected if the firm inappropriately evaluates the required level of production and manufacturers more than it can sell in the market.

Effective capital of the firm is equal to

$$K_i(t) = \mathrm{QS}_i(t)/A(r). \tag{39}$$

Global sales are equal to

$$\mathrm{GS}(t) = \sum_i \mathrm{QS}_i(t) p_i(t). \tag{40}$$

The market share of firm-unit $i$ in global sales is equal to

$$\mathrm{fs}_i(t)=\mathrm{QS}_i(t)p_i(t)/\mathrm{GS}(t). \tag{41}$$

## 3. The model as a didactic and research tool

The model presented in the previous section has been programmed in TURBO PASCAL 5.5. The computer program is designed to serve as a tool to investigate theoretical problems of industrial development and as a didactic tool in teaching industrial economics.

The program contains a number of menus to choose relevant actions. There is a possibility of changing the values of the model's parameters both before starting a simulation run and during a simulation. It is also possible to play a 'manager game' with the participation of a number of players playing against the computer. The manager game (as we call this option) and the possibility of changing values of the model's parameters enable students to observe how interconnected and interdependent the mechanisms and relationships in the simulated process of industry development really are.

At any moment of cooperation with the program there is the possibility to see the results of simulation in numeric and graphic forms. It is also possible to see results of simulations made in the past.

As mentioned the program may serve as a research tool. In the option *Change of parameters' values*, besides a number of parameters related directly to the model there are parameters which govern the structure of the model and allow distortions of the natural mechanisms of development to be introduced. The structure of the program has been designed in such a way that it is possible to add new mechanisms and new processes in the future development of the model (one of the possible directions of development is simultation of co-evolution of a number of industries and markets).

## 4. Simulation

In this section some preliminary results of simulation of the model are presented. Results of systematic study of the model, impact of different parameters on behavior of the model, study of factors of concentration of industry, simultation of the model in some interesting artificially created situation, etc. will be published in the future.

### *4.1. How are prices set?*

The problem which seem interesting to us and also to be solved before future systematic study of the model is 'how are prices set?'. We have assumed that the price in the decision making procedure of a firm [eq. (10)–

(21)] is set by a firm by using some 'external' procedure. Production and investment in the next year are estimated on the base of this price.

The problem 'how are prices set' is discussed by Silverberg (1987). He rejects the marginalist theory as well as monopoly price/output criterion (marginal revenue equal to marginal cost) as inapplicable to his model. Silverberg proposes difference equations for price dynamics on the base of the markup theory: 'firms determine average unit cost at some standard operating capacity, and add a fixed percentage to arrive at a price which is otherwise independent of demand and competitive pressures.' His dynamic adjustment equation 'capture the main aspect of the problem, and at the same time allows for shift in the price structure due to long-term changes in relative cost competitiveness' [Silverberg (1987)].

Silverberg uses continuous time in his model. To adjust his formula to our model we have to reformulate it as a system of difference equations (we use discrete time). Using our notations the parallel price dynamic equations applied in our model has the form:

$$p_i(t+1) = p_i(t)[1 + a_1 \ln(a_3 V/p_i(t)) + a_2 \ln(c_i/c^e)], \tag{42}$$

where $p_i(t)$ is the price of products of firm $i$ at time $t$, $a_1$, $a_2$, $a_3$ are parameters, $V$ is the cost of production (variable and constant), $c_i$ the competitiveness of products of firm $i$, and $c^e$ the average competitiveness of products on the market.

We agree that Silverberg's equation 'capture the main aspect of the problem' and reflects what some firms are doing. Our model enables us to compare effectiveness of different price setting procedures (PSP) and observe how the distribution of firms using different PSP changes in the course of development of the industry. What we propose is to invent different price setting procedures and allow each firm to choose (e.g., randomly) the price setting procedure applied by this firm during its lifetime. By making a number of such experiments we are able to evaluate which PSP wins in such a game and which class of behavior is more likely to allow them to co-exist or to dominate the market. We treat the proposition of Silverberg as one of these PSPs (rules). The others which we investigate are based on the assumption that a firm evaluates (estimates) values of some objective function for different values of the price of its production. The price for which the objective function is maximized is chosen by a firm as the price of its products. It is not maximization in the strict sense. The estimation of the values of the objective function is not exact and made only for the next year (this is not global, once and for all, optimization, since firms apply this rule from year to year). We propose two objective functions:

$$O_1(t+1)=(1-F_i)\Gamma_i(t+1)/\Gamma(t)+F_iQ_i(t+1)/\mathrm{QS}(t), \tag{43}$$

$$O_2(t+1)=(1-F_i)\Pi_i(t+1)/\Pi(t)+F_iQ_i(t+1)/\mathrm{QS}(t), \tag{44}$$

$$F_i=a_4\exp(-a_5Q_i(t+1)/\mathrm{QS}(t)), \tag{45}$$

where $F_i$ is the magnitude coefficient, $Q_i$ the expected production of firm $i$ in year $t+1$, $\Gamma_i$ the expected income of firm $i$ at $t+1$, defined by eq. (20), $\Pi_i$ the expected profit of firm $i$ at $t+1$, defined by eq. (21), QS the global production of the industry in year $t$, $\Gamma$ the global net income of all firms in year $t$, and $\Pi$ the global net profit of all firms in year $t$.

If we assume that $a_4$ is equal to zero then the objective of a firm is simply net income in the first case, and profit in the second case. For $a_4$ greater than zero the objective of a firm is in some sense a combination of short-term thinking (search for profit or income in the next year) and long-term thinking (obtaining a larger market share in the future). The parameter $a_5$ controls the variation of magnitude of short- and long-term thinking for small and large firms. For $a_5$ greater than zero the long-term thinking is more important for the small firms than for the large firms. We add this parameter to check if it is essential for firms with larger market shares to change their strategies or to stick to a rigid strategy (as defined by parameter $a_4$).

We use a formal procedure to find the price and we treat this procedure as an approximation of what is made by decision makers in reality. They, of course, do not make such calculations from year to year; rather they think in a routine mode: 'My decisions ought to provide for the future prospects of the firm and also should allow profit (or income) to be maintained at some relatively high level.'

The values of parameters $a_1$, $a_2$, $a_3$, $a_4$, and $a_5$ for each firm are randomly drawn from defined sets of values for each firm in the year of its entry into the market. This gives an opportunity to observe the competition of firms applying the same rule for different values of relevant parameters as well as competition between firms applying different rules. We have made the following series of experiments:

(1) All firms entering the market apply the same price setting procedure (the markup rule, the $O_1$ rule, or the $O_2$ rule). Values of relevant parameters are randomly drawn for each firm.

(2) At the time of entry a firm chooses randomly one of two (or three) price setting procedures (there are four combination: (a) Markup and $O_1$; (b) Markup and $O_2$; (c) $O_1$ and $O_2$; and (d) All three price setting procedures.)

The initial conditions in all experiments are the same (see Appendix A). We have repeated these experiments for two values of cost of production (assumed to be constant during simulation) i.e., equal to 1.3 and 2.6.

In the markup rule values of parameters $a_1$, $a_2$ and $a_3$ are drawn from sets (0, 1), (0, 0.5), and (1, 3), respectively. The results of 20 simulation runs for each cost of production for the markup rule are summarized in fig. 1 (*a* and *c*). Coordinates $a_1$ and $a_3$ for the largest firms (i.e., firms with market shares in the last year of simulation ($t=100$) greater than 4%) are marked by rectangles).[3]

If all firms which enter the market in these simulations have been marked in these pictures then the distributions of these firms would have been uniform in the rectangle defined by the minimum and the maximum values of parameters $a_1$ and $a_3$. The distribution of the largest firms is far from the uniform distribution. Average values of parameters $a_1$, $a_2$, and $a_3$ for the largest firms are equal to: 0.364, 0.132, and 1.939 (for $Cost=1.3$), 0.357, 0.138, and 1.489 (for $Cost=2.6$), respectively. These points are marked by the crosses in fig. 1, the lengths of arms of the crosses are equal to standard deviations of $a_1$ and $a_3$. The values of $a_1$ and $a_2$ are not essential for firms' survival, values of these parameters are almost uniformly distributed within the scope of their variablity. But values of $a_3$ are crucial for firms' survival. Standard deviation of this parameter is relatively small.

Similar experiments were made for both objective functions. The results for the $O_1$ rule are presented in fig. 1b and 1d. As in the experiments with the markup rule the largest firms are marked within the rectangles defined by the minimum and the maximum values of parameters $a_4$ and $a_5$ (we assume that $a_4$ varies between 0 and 1, and $a_5$ between 0 and 10). Average values of parameters $a_4$ and $a_5$ are equal to 0.783 and 4.26 (for $Cost=1.3$); 0.803 and 4.52 (for $Cost=2.6$), respectively. Parameter $a_5$ is almost uniformly distributed within assumed borders, which means that $a_5$ is not as essential for firms survival as $a_4$. The average value of $a_4$ suggest that long-term thinking is decisive for firms' survival. Reaching high income (or profit) is important but considerably more important is concern with the future of a firm.

Average values of the parameters $a_4$ and $a_5$ are almost the same for different costs of production. Pictures in figs 1b and 1d are very similar. This implies that firms may apply the same $O_1$ rule for different costs of production (experiments were made for the $O_1$ rule under different initial conditions and the results suggest that the 'optimal' $O_1$ rule is invariant for different simulation conditions).

The situation is different for the markup rule, where the average values of parameters $a_3$ for two levels of the cost of production are significantly

[3]We ought to present the results in a three dimensional space $(a_1, a_2, a_3)$ but to make the picture readable we confine the presentation to the plane $(a_1, a_3)$. The distribution of the parameter $a_2$ is almost uniform; i.e. is similar that that of $a_1$.

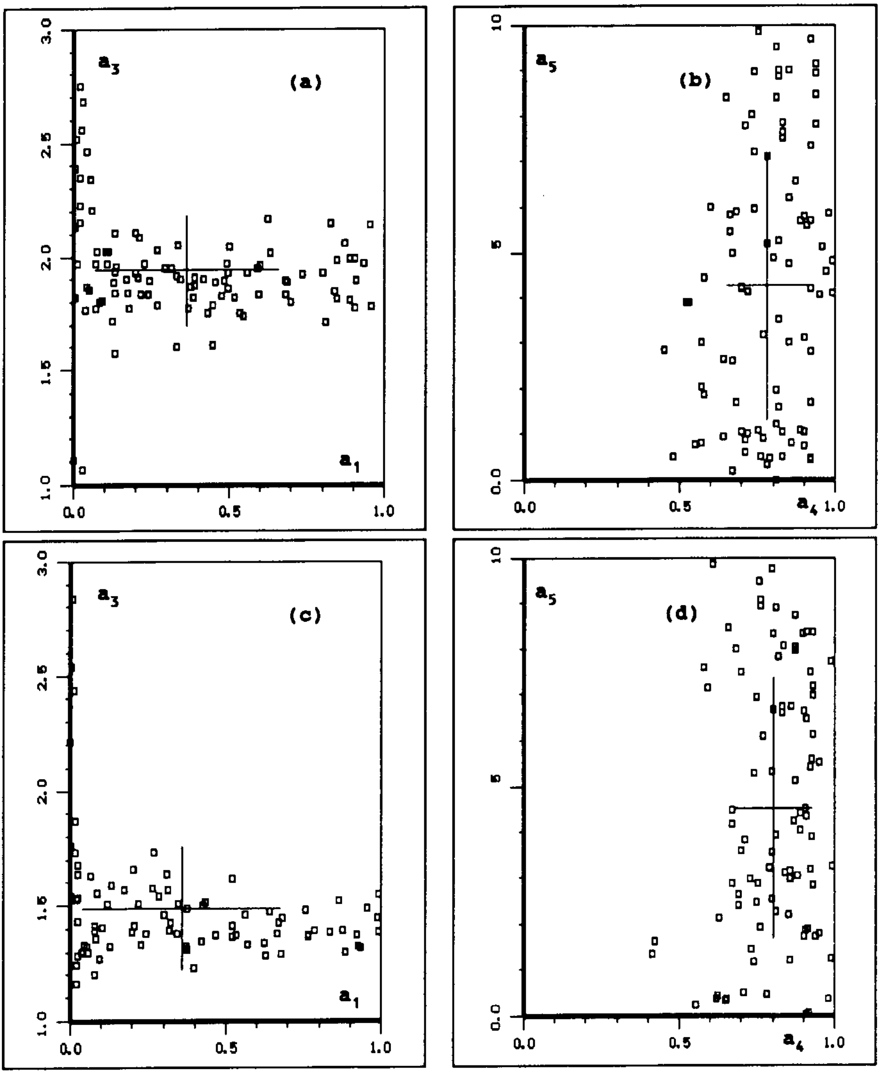


Fig. 1. Coordinates of the parameters in the price setting procedures for the largest firms; the markup rule (a and c), and the $O_1$ rule (b and d), for two levels of the cost of production: $Cost = 1.3$ (a and b), $Cost = 2.6$ (c and d).

different (1.94 and 1.49, respectively). This suggests that the 'optimal' markup rule is different for different simulation conditions.

The following experiment implies that, contrary to the markup rule, the $O_1$ rule is invariant for different states of the industry. For given simulation condition (see Appendix A) and the variable cost equal to 2.6, the parameters

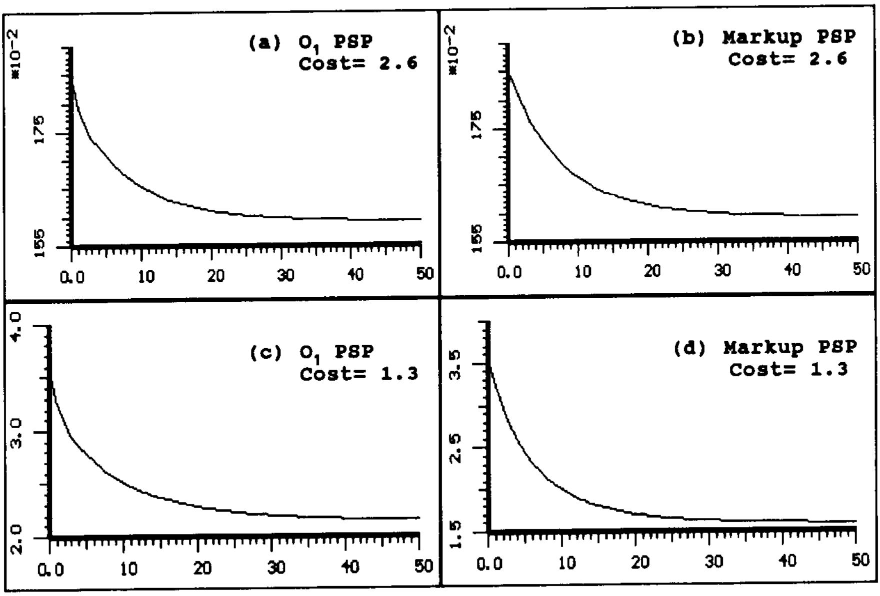


Fig. 2. Price to cost ratio for the $O_1$ rule and the markup rule, for two values of the cost parameter.

$a_4$ and $a_5$ of the $O_1$ rule and parameters $a_1$, $a_2$, $a_3$ of the markup rule are estimated to get in both cases similar development of the industry. The values of the parameters were: $a_1=0.115$, $a_2=0.05$, $a_3=1.5928$, $a_4=1$, and $a_5=5$. The differences between these two runs are negligible; the characteristics of global development are almost the same (see e.g. the price/cost ration presented in fig. 2a and 2b). The equilibrium price is equal to 4.3 and the equilibrium value of the price to cost ratio is equal to 1.5928 in both experiments. Small (but negligible) differences existed for some other characteristics, e.g., the profit to capital ratio was 1.015% for the $O_1$ rule and 1.016% for the markup rule.

In the next two runs the cost of production was diminished to 1.3. Firms which apply the $O_1$ rule adjust to the new conditions easily, the new equilibrium price is smaller than in the previous run (3.0) but the price/cost ratio is greater (2.14). The values of the global characteristics at equilibrium state are very similar in both runs, e.g., the profit/capital ratio is equal to 1.015% for $Cost=2.6$, and 1.017% for $Cost=1.3$ (fig. 3a and 3c). The average price in the second run with the $O_1$ PSP is smaller than in the previous experiment so the global production is larger in this run (fig. 3b and 3d).

The adjustment to the new condition is not observed in a similar run with firms applying the markup rule. Because of the stable value of the $a_3$

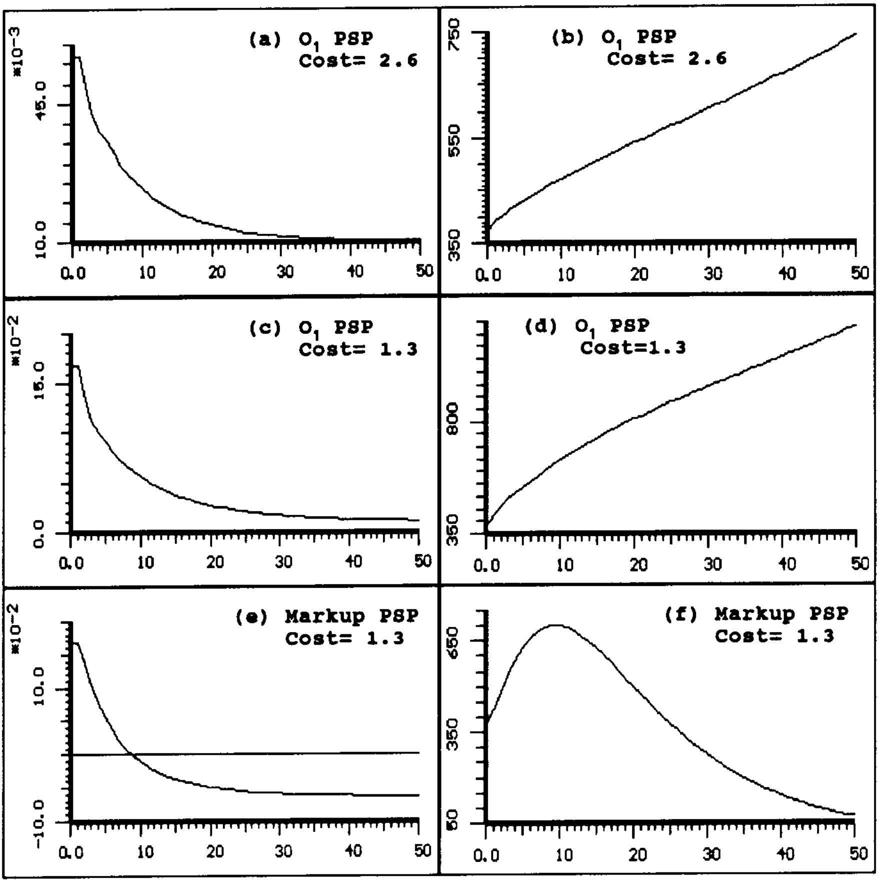


Fig. 3. Profit to capital ratio (a, c, and e) and the global production (b, d, and f) for the $O_1$ rule and the markup rule for two values of the cost parameter.

parameter, the price in this case is lower than in all previous runs (equal to 2.23), the equilibrium price/cost ratio is the same as in the previous experiment with the markup rule (equal to 1.593). Because of the low price the profit of the firms falls below zero (fig. 3e), firms have no funds to invest (the investment/capital ratio is equal to 3.5%, i.e. much less than the capital physical depreciation, which is 10%) and global production also falls (fig. 3f). To get similar results for the markup rule (as for the $O_1$ rule) in the case of the reduced cost it is necessary to adjust the markup parameters (e.g., $a_3$ ought to be equal to 2.14). So to get plausible results it would be necessary to incorporate into the model some meta-rule for adjusting the markup parameters accordingly to the changing state of the industry.

Table 1

Long- and short-term objectives in the decision making pricess ($a_5=0$). Global characteristics in years 20–100.

| $a_4$ | $n_H$ | $\Pi/K(\%)$ | $\Pi^*/K(\%)$ | $\Pi/S(\%)$ | $I/K(\%)$ | $p/V$ |
|---|---|---|---|---|---|---|
| 0.0 | 11.65 | 39.647 | 38.627 | 37.290 | 11.020 | 2.063 |
| 0.1 | 11.68 | 29.418 | 28.403 | 30.616 | 11.015 | 1.865 |
| 0.2 | 11.70 | 20.988 | 19.976 | 23.950 | 11.012 | 1.701 |
| 0.3 | 11.72 | 13.919 | 12.907 | 17.272 | 11.012 | 1.564 |
| 0.4 | 10.61 | 8.035 | 6.031 | 10.793 | 12.004 | 1.460 |
| 0.5 | 10.56 | 2.932 | 1.698 | 4.213 | 11.234 | 1.353 |
| 0.6 | 8.89 | 1.021 | 0.010 | 1.508 | 11.011 | 1.314 |
| 0.8 | 8.89 | 1.020 | 0.009 | 1.507 | 11.011 | 1.314 |
| 1.0 | 8.89 | 1.020 | 0.010 | 1.507 | 11.010 | 1.314 |

Hundreds of simulation runs with the $O_1$ rule with different simulation conditions (different number of competitors, different values of the model parameters such as $\alpha$, $\beta$, $\mu$, $\gamma$, and so on) show that the industry adjusts to these conditions without changing the parameters $a_4$ and $a_5$. If we vary the values of $a_4$ between 0.6 and 1.0 the development of the industry does not change significantly (see the results presented in the next section). This means that the development of the industry is not sensitive to values of $a_4$ (for $a_4$ greater than 0.6) and it confirms our conjecture that firms need not apply the $O_1$ rule in such an analytical form as implemented in the model, i.e., in reality the $O_1$ rule take on the form of a general routine: 'Make decisions that provide for future prospects of the firm and at the same time allow income to remain at some relatively high level.'

The possibility of choosing all combinations of the price setting procedures was allowed in a number of experiments (for each combination at least 15 simulation runs were made). There is no place to present in full detail the results of these experiments. The general findings are as follows: The $O_1$ rule wins against either the markup rule and the $O_2$ rule; the markup rule always wins against the $O_2$ rule. Similarly, the $O_1$ rule beats the markup rule and the $O_2$ rule in experiments with all three price setting procedures available.

To check to what extent the $O_1$ rule is better than the markup rule, the following experiments were made. During the first 20 years all firms choose only the markup rule, and after $t=20$ entering firms choose, with the same probability, either the markup rule or the $O_1$ rule. In spite of the initial advantages of the markup rule over the $O_1$ rule firms which apply the $O_1$ rule dominate the market at $t=100$ in all these experiments. Using the biological terminology, it may be said that the $O_1$ rule is an evolutionary stable strategy [Maynard Smith (1982)].

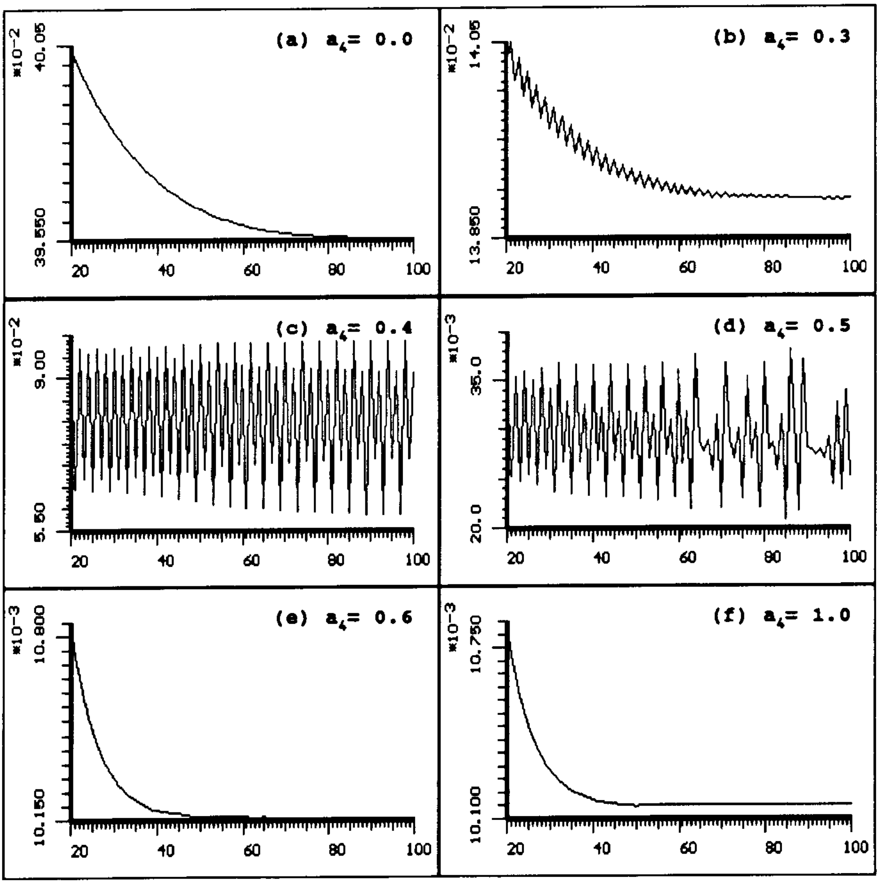


Fig. 4. Profit to capital ratio for different relative importance of long- and short-term objectives in firms' decision making process.

### *4.2. Long- and short-term objectives in the decision making process*

For the $O_1$ rule two series of experiments were made; in the first series firms apply an inflexible stategy, i.e., the parameter $a_5$ is equal to 0 and only parameter $a_4$ is changed. In the second series firms apply a flexible stategy, i.e., the parameter $a_5$ is changed and the parameter $a_4$ is fixed (i.e., is equal to either 0.4 or 1.0).

The results of the first series of experiments are presented in table 1,[4] and

[4]The symbols used in the tables are as follows:
$n_H$ average value of the Herfindahl firms' number equivalent;
$K$ average capital;
$Q$ average production;

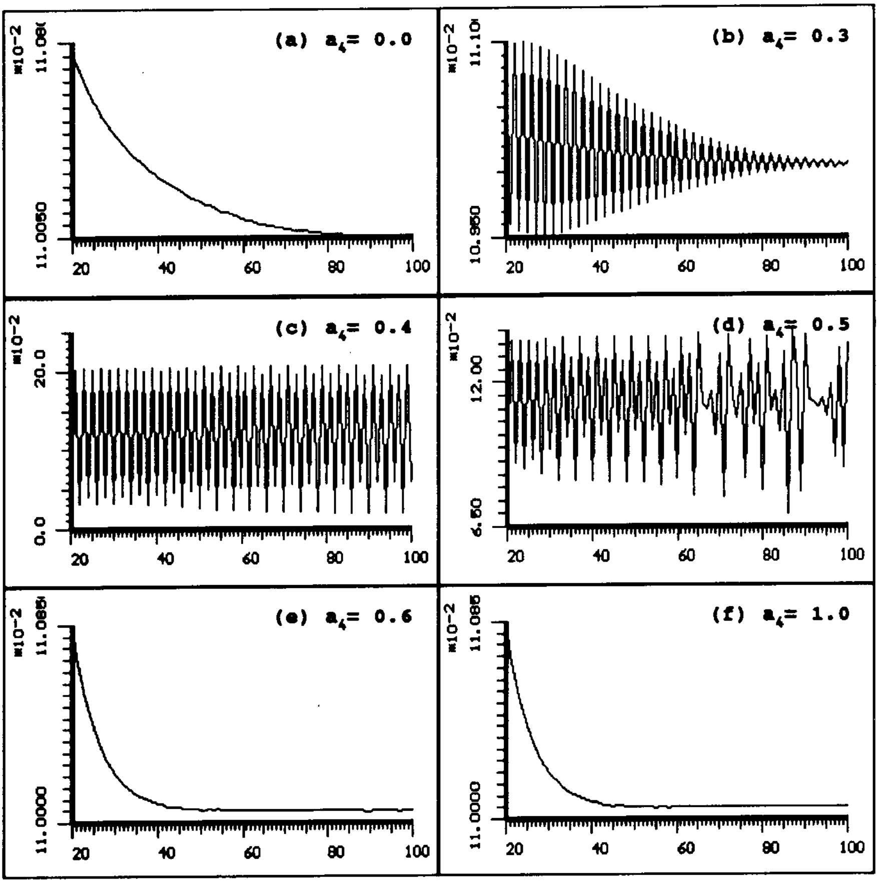


Fig. 5. Investment to capital ratio for different relative importance of long- and short-term objectives in firms' decision making progress.

in figs. 4 and 5. Values of the profit/capital and profit/sales ratios are very high (around 40%) for $a_4$ close to zero, i.e., the short-term objective (the income) dominates; they diminish to a rather small value (slightly over 1%)

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$V$ average cost of production;
$I$ average investment;
$\Pi$ average profit;
$\Pi^*$ average discounted profit; $\Pi^w = \Pi - (T - \delta K)$, where $\delta$ is the capital depreciation rate.

The Herfindahl–Hirschman index of concentration of the industry is equal to

$$H = \Sigma_i (f_i)^2,$$

The Herfindahl firms' number equivalent is defined as

$$n_H = 1/H,$$

and is the number of equal-sized firms that would have the same $H$-index as the actual size distribution of firms.

Table 2

Flexible strategies in the decision making process. Global characteristics in years 20–100.

| $a_5$ | $n_H$ | $\Pi/K(\%)$ | $\Pi^*/K(\%)$ | $\Pi/S(\%)$ | $I/K(\%)$ | $p/V$ |
|---|---|---|---|---|---|---|
| $a_4=0.4$ | | | | | | |
| 0.0 | 10.61 | 8.035 | 6.031 | 10.793 | 12.004 | 1.460 |
| 1.0 | 11.87 | 9.754 | 8.763 | 12.763 | 11.013 | 1.483 |
| 2.0 | 11.93 | 11.524 | 10.509 | 14.737 | 11.015 | 1.519 |
| 3.0 | 11.96 | 13.219 | 12.203 | 16.547 | 11.016 | 1.551 |
| 5.0 | 11.99 | 16.397 | 15.383 | 19.740 | 11.015 | 1.613 |
| 10.0 | 12.00 | 23.118 | 22.113 | 25.749 | 11.006 | 1.743 |
| $a_4=1.0$ | | | | | | |
| 0.0 | 8.89 | 1.020 | 0.010 | 1.057 | 11.010 | 1.314 |
| 1.0 | 8.90 | 1.020 | 0.009 | 1.507 | 11.011 | 1.314 |
| 3.0 | 8.89 | 1.020 | 0.009 | 1.707 | 11.011 | 1.314 |
| 5.0 | 9.61 | 1.115 | 0.076 | 1.644 | 11.039 | 1.314 |
| 7.0 | 10.99 | 2.280 | 1.169 | 3.298 | 11.111 | 1.325 |
| 10.0 | 11.84 | 6.602 | 5.498 | 9.002 | 11.104 | 1.415 |

for $a_4$ close to one, i.e., when the long-term objective dominates. The discounted profit rate for $a_4$ greater than 0.6 is 0.01%.

There is some kind of unstable behavior for values of $a_4$ between 0.35 and 0.55, i.e., when both objectives are almost equally important. For this setting the behavior of the firm is apparently chaotic, with fluctuations of the profit/capital ratio of the same order as the mean of this ratio. The same kind of behavior is observed for other characteristics of industry development, e.g., the investment to capital ratio (fig. 5c and 5d). The fluctuations of the investment/capital ratio are so high that frequently the values of this ratio drop below the 10% (i.e., below the value of capital physical depreciation) and the global production is significantly reduced.

The chaotic mode of development of the industry is not observed if firms apply a flexible strategy, i.e., the value of $a_5$ is greater than zero (see table 2). It may be said that the $a_5$ parameter acts as a filter. The changes of the global characteristics are smooth, as in figs. 4a, 4e, 4f and 5a, 5e, 5f. As we compare the values of the relevant characteristics of development in tables 1 and 2 we see that parameter $a_5$ shifts the characteristics towards the smaller values of $a_4$, and the greater the value of $a_5$ the more significant this shift is; e.g., the characteristics for $a_4=0.4$ and $a_5=3.0$ are similar to those for $a_4=0.3$ and $a_5=0.0$.

It seems that in real processes of industrial development firms change the relative importance of the long- and short-term objectives in the course of their development. The long-term objective is much more important in the first phase of a firm's development. If firms reach significant share in global production the short-term objectives becomes more important. As the simulations suggest the chaotic kind of development is not observed if firms use the flexible strategy, it may be expected that real processes are far from the chaotic mode of development. However short periods of chaos in industrial development cannot be excluded.

## Appendix A. Initial states of the simulation experiments

In all simulation runs presented in the paper the following values of the parameters are assumed: Number of segments is 5; number of technical characteristics ($m$) is 7; number of active routines is 50.
Research funds proportional to the firm's capital [eq. (1)].

$$h_1 = h_2 = 0, \quad h_0 = 0.005 \text{ or } h_0 = 0 \text{ for no innovation.}$$

R&D strategy [imitation, innovation – $G$ in e.q. (3)] is 100.
No. of experiments in R&D process [eq. (4)]; $e=0$ if no innovation, otherwise $e=10$, $\psi=0.5$, $E=0$.
Probability of mutation [eq. (5)]; $a^{\mathrm{m}}=0.007$, $\zeta=0.5$, $b^{\mathrm{m}}=0$.
Probability of recombination [eq. (6)]; $a^{\mathrm{r}}=0.03$, $\xi=0.5$, $b^{\mathrm{r}}=0$.
Probability of recrudescence [eq. (7)]; $u_1=1$, $u_2=0.005$.
Price elasticity in competitiveness [eq. (9)]; $\alpha=2$.
Prediction of price and comptitiveness [eqs. (12), (13)]; $\sigma=0$.
Initial size of the market [eq. (15)]; $N=40$.
Growth rate of market size [eq. (15)]; $\gamma=0.01$.
Elasticity of price in demand function [eq. (15)]; $\beta=-0.3$.
Economy of scale; $\varepsilon=0$.
Constant production cost [eq. (21)]; $\eta=0.10$.
Normal rate of return [eq. (21)]; $\rho=0.05$.
Capital physical depreciation [eqs. (21), (23)]; $\delta=0.10$.
Modernization investment [eq. (22)]; $k_1=k_2=k_3=4$.
Investment capability [eq. (23)]; $\mu_0=1.0$, $\mu_1=10{,}000$.
Initial capital (for entrants and new units); $InitCapital=10$.
Range of routines, MinRit = 0, MaxRut = 64,000.

The transformation functions of routines into the technical characteristics [eq. (8)], productivity of capital $A(r)$, and variable cost of production $V(r)$ are assumed to be linear.

$$z_d = \sum_i c_{di} r_i \quad \text{for } d=1,2,\ldots,m.$$

$$A(r) = \sum_i a_i r_i, \quad V(r) = \sum_i v_i r_i.$$

There is no place to present all values of coefficients $c_{di}$, $a_i$ and $v_i$. The values were initially selected to yield plausible constants in the initial state of the industry.

The technical competitiveness function $q(z)$ – eq. (9) – is

$$q(z_1, z_2, \ldots, z_m) = \exp\left( -0.00125 \sum_i (z_i - z_i')^2 \right).$$

The function $q$ has the shape of a hill with the maximum equal to 1 and the distance of each optimal coordinate $z'$ from the initial state equal to 12.

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