Kitchin, Juglar and Kuznetz business cycles revisited[1]

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Abstract

The simulation model of business cycles emergence, due to the natural delay between investment and production ('delayed response effect'), is presented. The main questions stated in the paper are following:

- What are necessary conditions for business cycles emergence? Does a delayed response effect is the only necessary factor causing cycles emergence? Are there other necessary factors which coincidence with delayed response effect cause cycles emergence?
- How cycles length depends on such factors as delay duration, type of the market, decisions modes, 'savings', etc.?

In the first section a preliminary model of well known hog's cycle is presented. The hog's cycle is used as a metaphor for more general model of business cycle, which is presented in the second section. The computer simulation of the model reveals the investment delay is really important factor but it is hardly to say that it is the only factor and even not the most important one. Emergence of fluctuations requires a tuning of different factors, namely the investment delay, the price delay, sensitivity of producers on signals flowing from the market, the growth rate of the market, the capital productivity growth rate, capital depreciation rate, elasticity of demand.

The Kitchin 3-5 years cycles emerge for the investment delay of the order 0.3 to 0.6 years and the price delay between 0.2 and 0.3 years. For higher investment delay (one to two years) and the price delay (one to 1.5 years) we observe the Juglar 7-11 years cycles. The Kuznetz infrastructural investment cycles (14-25 years) are observed for the investment delays between 3 to 5 years.

Fluctuations and cyclical behaviour are observed in majority of socioeconomic processes; the literature on this subject is enormous and has its own long history. It is impossible to make any review of literature in a short paper therefore we confine the introductory remarks to state only the main points on former work done by different authors related to the presented model. The four early authors working on empirical and theoretical evidences of business cycles are: Clément Juglar who in the middle of the 19th century tried to proof existence of 8-10 year business cycles, Nikolai D. Kondratieff who investigated existence of so called Long Waves of 50-60 years longevity in the beginning of the 20th century, Joseph Kitchin, who at the same period searched for short, roughly 3 years, inventory cycles, and Simon Kuznetz who in 1930 noticed 15-25 years length cycles and associated them with fluctuations in rates of population growth and immigrating but also with investment delays in building, construction, transport infrastructure, etc. (Juglar, 1862; Kitchin, 1923, Kondratieff, 1935; Kuznietz, 1930). It was Schumpeter who in his *Business Cycles* called the three cycles as 'Juglar', 'Kondratieff' and 'Kitchin', respectively and noticed that "[b]arring very few cases in which difficulties arise, it is possible to count off, historically as well as statistically, six Juglars to a Kondratieff and three Kitchins to a Juglar – not as an average but in every individual case" (Schumpeter, 1939, pp. 173-174). For some reasons, Schumpeter have not included into his considerations the Kuznetz investment cycles.

The range of proposed factors causing business cycles is really wide: from climate, over-investment, monetary, underconsumption to psychological factors. Naturally it would be difficult to include all those factors in one model.[2] The presented model does not relate directly to any distinguished schools explaining business cycles emergence (e.g., Climate Theories of the Cycle, Over-Investment Theories, Psychological and Lead/Lag Theories, Monetary Theories of the Cycle, Underconsumption Theories[3]), although to some extend the model is related to the ideas of Albert Aftalion, Arthur C. Pigou and John M. Clark who assumed that expectations ought to be considered as the central cause of fluctuations. Albert Aftalion (1909, 1913) proposed that investment expansions are not based only on real factors such as "technological change" and an "abundance of loanable funds", but rather on the *expectation* by businesses of consumer demand and profits. According to Aftalion, firms make investment decisions when demand is high. The downswing arises, Aftalion concludes, when the investment projects are finished, generally around the same time, and a wave, a "superproduction" of consumer goods begins to come out of the newly-built factories. Arthur C. Pigou (1920, 1927) continued this work and stressed the psychological factor, mainly related to expectations of profits as the driving factor of investment. In opinion of Pigou, these expectations can arise easily from errors and miscalculations of entrepreneurs. These errors can lead to large rises or falls in investment and this is what generates cycles.

[1] The work have been partly done within the Dynamics of Institutions and Markets in Europe (DIME) Project (http://www.dime-eu.org/).
[2] Although we are working on a general model of national economy where most of those factors will be included. The first results of computer simulation of that model are very promising.
[3] e.g., http://cepa.newschool.edu/het/essays/cycle/cyclecont.htm

The presented model relates also to diversified stream of economic research on cycles emergence stressing the importance of delayed response effects, i.e., relates to systems where the future development is dependent on the state of the system in the more or less distant past. There are numerous causes and effects considered, e.g. interest rate changes cause changes of capital with a delay of roughly 6 to 18 months, business profitability causes changes of production capacity with delay of 1 do 2 years, price causes changes of demand with delays of few weeks to two years, salaries influence availability of workers with delay of 3 to 8 years, credit availability influences the increase of loans (especially bad ones) with delay of 1 do 5 years.

In our opinion the proper explanation of business cycles emergence require development of nonlinear models. Our proposition is in accordance to Richard Goodwin's (1951) approach, who was one of first researchers insisting use of nonlinear dynamical systems in business cycle theory to generate endogenous fluctuations. It is common to recognize three phases in Goodwin's theorizing of the macroeconomic fluctuations, corresponding to, respectively: the *limit cycle* of the 1950s, the Lotka-Volterra *orbital oscillator* of the 1960s and *chaotic attractors* in the later decades. We are not pursuing along the research route proposed by Goodwin but we agree that proper explanation of business fluctuations and cycles ought to be based on nonlinear models, which are useful to identify necessary conditions for emergence of closed cycles (constant oscillations), and show that for wide range of initial conditions, economic system goes to closed cycles and that the explosive oscillations would not have been observed in the model (this property seems to be crucial to evaluate the reality of the model because as we can expect, due to existence of control mechanisms, the emergence of explosive oscillations (with amplitude going to infinity) is very improbably in real economic processes).

The main aim of the paper is to show how business cycles emerge due to the natural delay between investment and production ('delayed response effects'). The problem is well known in literature but we hope to shed new light on that classical problem. The proposed model is not a macroeconomic model in which usual aggregates as GNP, Consumption, Investment, Aggregate Demand, Aggregate Supply, Government spending, Taxation, etc. are applied. The model ought to be considered as a model of the leading industry causing cycles emergence of national economy. This approach was used e.g. by Mikhail I. Tugan-Baranovsky (1894) who investigated output fluctuations of the production of iron, and use it as an indicator of national business cycle emergence. He argued that expansions were derived from sudden and massive finance-induced spurts of investment.

The main questions stated in the paper are following:

- What are necessary conditions for business cycles emergence? Does a delayed response effect is the only necessary factor causing cycles emergence or are there another necessary factors which coincidence with delayed response effects causes cycles emergence?
- How cycles length depends on such factors as delay duration, type of the market, decisions modes, ‘savings’, etc.?

It seems that the good starting point is to build a preliminary model of well known hog’s cycle and use that model as a metaphor for more general model of business cycle. The model of hog’s cycle and its simulation results are presented in the first part of the paper. The model tries to reflect real process of hogs breeding, e.g., the model’s parameters settings are done in the accordance to real biological processes.

In the second section of the paper the general model of investment business cycle is presented. The formal description of the model is followed by presentation of results of its computer simulation. As it was mentioned, the model is nonlinear one and probably it is not possible to find analytical solution of the model. Therefore a simulation approach of the models investigation was used. This allows for much more flexible investigation of the models properties; for example, in the context of business cycles modelling, very rarely a question of impact of growing economy on cycles emergence is stated (namely to what extend a growth ratio of market capacity influences the mode of economic development). The simulation of the model is focused on answering the presented above questions but also on investigation of general properties of the model.

The hog’s cycle model revisited

Frequently used example illustrating emergence of business cycles caused by existence of natural delays is so called hog’s cycle. Idealized story behind that cycle is following: farmers raising hogs observe a market price of pork and compare it to the current costs of production. While the price is rising and the more the current price is above the cost of production, more and more farmers decide to increase the hogs stock. Natural delay between the decision on increasing the number of hogs and the moment of selling the pork (slay a porker) is around nine months (114 days of gestation and six month of breeding). After that period a supply of pork on the market is increasing. Due to market price setting mechanism, high supply causes declining the price of pork sold on the market. In a course of declining prices, incentives for hogs’ breeding diminish also. Therefore farmers delimit the stock of hogs and after some period a supply of pork is declining. Naturally due to declining supply the pork price is increasing, so the cycle starts again. As an example of such process a number of hogs and sows in Poland are presented in Figure 1.

A pork price (p) sold on the market is governed by market mechanisms and depends mainly on the discrepancy (r) between the pork demand (D) and a potential supply of pork (i.e., a number of porkers). The price is increasing while demand surpasses the supply and is decreasing in the opposite case. The process of price setting can be described by a difference equation:

$$\frac{dp}{dt} = \frac{r \cdot p}{\delta}, \qquad (1)$$

where δ is a delay between a decision on price adjustment and its effective use in the market. Discrepancy between supply and demand r is defined as follows:

$$r = \frac{D - (1 - sf) \cdot n \cdot w}{D}, \qquad (2)$$

where:

D – a pork demand (in kg);

n – a number of hogs (i.e. porkers and sows potentially to be slew and sold on a market as a pork);

sf – share of sows in total number of hogs (so called sows fraction);

w – average weight of a porker.

The demand D can be modelled by a classical demand function with a constant price elasticity e (using the demand function with constant price elasticity allows to investigate the influence of a different types of a pork market on a mode of hogs breeding process).

$$D = A \cdot (p / p_0)^e, \qquad (3)$$

where:

D – a pork demand (in kg);

e – price elasticity (we call a market elastic one if e is smaller then -1 and inelastic if e is greater then -1 and smaller then 0) ;

p – a pork price sold on the market;

p_0 – a reference price of a pork (the reference price is related to a cost of production, it is a border price for which the profit is equal to zero; using the reference price in the demand function allows to start the simulation of the model from the same initial condition for different values of elasticity e);

A – a parameter related to the initial size of the market.

A change of a number of hogs (n) is a result of balance between a number of piglets (m), born and bred to the stage of porker (taking into account the delay τ flowing from the gestation and the breeding period) and the slaughter (u):

$$\frac{dn}{dt} = m(t-\tau) - u(t) \tag{4}$$

A number of piglets (m) passing to the stage of porkers at time t is equal to a number of piglets born at time (t-τ), namely

$$m(t-\tau) = (pl(t-\tau) - u(t-\tau))(1 - e^{-ss(p(t-\tau)-p_0)/p_0}) + u(t-\tau) \tag{5}$$

where:

p_0 – the reference price, i.e. for current price grater then p_0 the pork production is profitable;

ss – parameter related to the 'sensitivity' of farmers to change a number of hogs accordingly to observed, current market price of a pork;

$u(t-\tau)$ – a slaughter at time (t-τ);

$pl(t)$ – breeding potential of sows at time t; it is equal to:

$$pl(t) = sf \cdot n(t) \cdot k\,, \tag{6}$$

where: sf is a fraction of sows (i.e. a number of sows related to total number of porkers), n is for number of porkers and k is an average number of piglets in a litter.

Let's notice that using equation (5) we are able to describe wide range of farmers' behaviour whose decisions depends on the market signals (especially related to the price of pork). For price p equal to the reference price p_0 number of piglets is just equal to number of slaughters (i.e., total number of hogs is not changing). The smaller the price (i.e., the greater the negative value of relative difference between the current price and the reference price) the smaller is the number of piglets, and vice versa the greater the price the more piglets are born and bred. Naturally the upper limit is breeding potential of sows (pl) which is accessed for very high price p.

A slaughter relates to current demand. If a number of porkers is so large that it is possible to cover current demand for pork (taking into account an average weight of a porker w) then relevant number of porkers are slew. If it is not possible to cover the current demand then maximal possible number of porkers is slew, therefore

$$u(t) = \min\{\frac{D}{w}, (1 - sf) \cdot n\} \tag{7}$$

The above simple model of hog's cycle was written in a convention of J.W. Forrester's *System Dynamics* approach. The flow diagram generated by *Stella*[4] is presented in Figure 2.

We will not present the detailed simulation study of that model and we will confine to present the results of the most basic simulation run.[5] It was assumed that the modelled process ought to reflect the process observed in Poland, therefore the values of the model's parameters are following: initial number of hogs – 15 millions, initial price is equal to the reference price p_0 = 3 Polish zloty, average weight of a porker w=130 kg, an average number of piglets in a litter k=10, a fraction of sows sf=0.15, gestation and breeding delay τ = 0,8 year, price elasticity e = -0,7, price adjustment delay δ =0.5 year, the 'sensitivity' of farmers to change a number of hogs ss=3.

Simulation results for those values of parameters are presented in Figures 3 and 4. Average length of the hog's cycle in that model is equal to 5.3 years. Two factors play important role in emergence of the cycles, namely the gestation and breeding delay (τ) and the price setting delay (δ).

Business cycle model

The hog's cycle model presented in the previous section can be used as a metaphor for building a more general business cycle model. Instead of thinking in specific terms such as porkers, piglets, gestation and breeding delay, progenitive potential of sows, slaughter, etc. we ought to think in more general terms of business process, namely capital, investment, investment delay, investment funds, production, etc. Some of the equations used in the hog's cycle model can be used almost directly, but some ought to be modified accordingly to general properties of a business process. Therefore we will add such characteristics of economic process as inventory, capital depreciation rate, capital productivity, etc.

Price setting equation is exactly the same as in the hog's cycle model. The price (p) of considered product (e.g. ships, buildings, TV sets) sold on the market is a result of natural market mechanisms and depends mainly on the discrepancy (r) between the demand (D) and a

[4] Stella is a simulation package developed by High Performance System ©.
[5] More detailed simulation study of hog's cycle model is presented in (Kwasnicki, 2002).

potential supply. The price is increasing while demand surpasses the supply and is decreasing in the opposite case. The process of price setting can be described by a difference equation:

$$\frac{dp}{dt} = \frac{r \cdot p}{\delta}, \tag{8}$$

where δ is a delay between a decision on price adjustment and its effective use in the market. The discrepancy is defined as the relative difference between the market demand (D) and the inventory (Inv), i.e. r is equal to

$$r = \frac{D - Inv}{D}, \tag{9}$$

where:

D – a product demand;

Inv – products' stock (inventory);

As in the hog's cycle model, the demand D is modelled by a classical demand function with a constant price elasticity e.

$$D = A \cdot (p / p_0)^e, \tag{10}$$

where:

D – the demand for products;

e – a price elasticity;

p – current product price sold on the market;

p_0 – the product's reference price;

A – a parameter related to the size of the market. It can be assumed that the market is growing with the rate γ, therefore we assume that A is a function of time (t) accordingly to the equation $A = A_0 \exp(\gamma t)$; where A_0 is the initial size of the market.

Changes of capital (C) engaged in the production is a result of balance between investment (I – taking into account the delay τ flowing from the natural time difference between the decision on investment and its effective use in the production process) and the capital depreciation (d):

$$\frac{dC}{dt} = I(t - \tau) - d(t) \tag{11}$$

Investment (I) used in the production at time t is equal to investment entering the investment process at time (t-τ), namely

$$I(t-\tau) = I_p(t-\tau)(1-e^{-ss(p(t-\tau)-p_0)/p_0}) \tag{12}$$

where:

p_0 – the reference price, i.e. for current price p grater then p_0 the production is profitable;

ss – a parameter related to the 'sensitivity' of producers to invest (i.e., to increase the production) accordingly to observed, current market price;

$I_p(t)$ – potential maximum of investment, equal to the current investment funds – investment funds are cumulative funds flowing from the adding current profit (or loss) from selling the production accordingly to the demand for production and the funds from the amortisation of capital used in the production process. The potential maximum investment fund is equal to:

$$\frac{dI_p}{dt} = S(t)(p(t)-p_0) + \rho C(t) \tag{13}$$

where $S(t)$ is current sale, $p(t)$ current price, p_0 reference price (unit cost of production), ρ is capital depreciation rate and $C(t)$ is capital currently used in the production. Therefore the expression $S(t)(p(t)-p_0)$ is current profit (or loss) and $\rho C(t)$ is current capital amortization (equal to the capital depreciation $d(t) = \rho C(t)$).

The price p above the reference price p_0 spurs the investment (eq. 12) and the higher the price the greater the investment is, but it does not exceed the current investment ability (i.e., potential maximum of investment). The price p below the reference price discourages entrepreneurs to invest and the smaller is the price the fewer entrepreneurs invest.

The inventory level (Inv) is a result of balance between the production and the sale. Production depends on used capital and its productivity (i.e., production is equal to multiplication of current productivity of capital $a(t)$ and the capital $C(t)$, i.e. $a(t)C(t)$) and the sale is equal to current demand D. Therefore

$$\frac{dInv}{dt} = a(t)C(t) - D(t) \tag{14}$$

The productivity of capital grows exponentially with the rate λ,

$$a(t) = a_0 \exp(\lambda\, t) \tag{15}$$

As in the case of the hog's cycle model the business cycle model has been written in a convention of Forrester's *System Dynamics* approach. The flow diagram generated by *Stella* is presented in Figure 5.

Simulation study of the business cycle model

The assumed values of the model's parameters do not reflect any real industry, their values has been selected just to be reasonable and to obtain plausible simulation results for so called base experiment. For that base experiment, initial value of the capital is equal to 100 units, inventory initial level is equal to 10 units, depreciation rate $\rho = 0.05$,[6] initial size of the market A_0 is equal to 10 units, the growth rate γ is equal to zero, price elasticity e=-0.7, investment delay τ is equal to 0.8 year, the price delay δ=0.5 year, the reference price p_0 is equal to 3 units, the 'sensitivity' of producers to invest *ss* is equal to 3, initial productivity of capital a_0=0.1, the capital productivity growth rate λ=0.[7] Results of the basic simulation run for the above values of the model's parameters are presented in Figure 6.

For the basic values of the model's parameters we observe the 7.8 years cycle fluctuations. Let us notice that due to the investment and the price delays there is a 'time shift' between the production and the sale of roughly 1.5 year.

The sensitivity parameter *ss* has very small influence on fluctuations at steady state (for small and large values of *ss* the cycles are around 8 years) but influences the mode of development in the 'transition period'. The smaller value of *ss* the longer the transition period (see Figure 7); for rather insensitive reaction of the firms on price signals (*ss*=0.2) in the first decades of development the process is seemingly approaching stable equilibrium but around 100 year the fluctuations emerge, and at the steady state the process is cyclical one; for *ss*=0.4 the steady state fluctuations (constant fluctuations) emerge earlier (roughly around 75 year). For large values of *ss* (*ss*=3.0 (Figure 6) and *ss*=7.0 (Figure 7c)) the transition period is very short one and the steady state fluctuations emerge in the first decade of development.

We can ask, to what extend the fluctuations depend on the delays? It turns out that for no price and investment delays (τ and δ are equal to zero) the process is going to the stable equilibrium – see Figure 8.

The arising question is: which one of the delays, namely investment delay (τ) or price delay (δ), is more essential for emergence of fluctuations. The duration of periods at steady

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[6] i.e., average lifetime of the capital is equal to 20 years.
[7] i.e., for the base experiment no technological advance is assumed.

state for different values of both delays is presented in Table 1. The simulations were done for 200 years because for some combinations of values the steady state emerged after a few decades from the initial moment of simulation runs (for that simulation run the moment of emergence of steady state fluctuations is indicated in Table 1 in parentheses). For small values of both delays (smaller then 0.1 years) there are no fluctuations (it is indicated by sign ∞ in Table 1). The longer the delay the longer period of fluctuations is. Analysis of numerous results of simulation runs (those presented in Table 1 are only small fraction of total runs made) suggests that both factors (i.e. investment and price delays) are almost equally important. Assuming one of the delays equal to zero, the other delay influences the fluctuation period almost in the same magnitude – the period changes from roughly 3.5 year for delay equal to 0.2 year, to 7 years for delay equal to one year, and to around 11 years for delay equal to 2 years. Although it is necessary to state that reasonable values of investment delays, observed in real processes, are much larger then the price delays. It seems that the one or two years price delays are very rarely (if ever) observed in real processes. The same can be said about the investment delays equals to 7 or ten years. We present the results for such long, seemingly unreasonable, delays just for purely theoretical reasons to show that thanks to possibility to make simulations it is possible to evaluate the model behaviour for created, although never observed in real processes, extreme states. It is worth to notice that for higher values of investment delay τ the fluctuation period does not depend significantly on values of price delay δ, e.g. for $\tau = 2$ years the period is almost the same,; for $\tau = 5$, changes are insignificant, namely the period is equal to 24.3 years for no price delay, 24.8 for 0.5 year price delay, and 25.9 years for $\delta = 2$ years.

What is also interesting, the amplitude of fluctuations is higher for greater values of investment and price delays, e.g., the amplitude of price fluctuations are around 0.15 for delay (investment or price) equal to 0.3 year, for 0.5 year delay the amplitude increases to 0.25, for τ=1 year and δ=0.5 year the amplitude is equal to 0.5, for τ=2 years and δ=0.5 year the amplitude is equal to 0.7.

The investment delay and the price delay are two main factors responsible for emergence of fluctuations but others model's parameters, such as capital depreciation ratio and capital productivity growth rate, influence also frequency (period) and amplitude of fluctuations. Although it is improbable that the capital depreciation ratio (ρ) is equal to zero, just to check how models behaves for that extreme value of ρ we run the model for ρ=0. It has occurred that very quickly, just after 6 years of transition period, the equilibrium is reached (see Fig.

9a). But for small values of ρ fluctuations are observed, e.g. for ρ=0.01 (i.e. the average lifetime of capital is equal to 100 years) the fluctuation period at steady state is equal to 8.1 years (see Fig. 9b), although the amplitude is not so high, around 0.1. For ρ= 0.05 (i.e., for base value of the parameter) the period of fluctuations are 7.8 years and the amplitude 0.45 (see Fig. 7). For larger values of ρ we observe the period shortening and the amplitude increasing, e.g., for ρ= 0.1, period is equal to 7.4 years, and the amplitude 0.75, for ρ=0.2 period is 6.8 years, the amplitude 1.4, for ρ=0.3 period is 6.5 years, amplitude 2.5, and for very high value ρ=0.5 (i.e. the capital is renewed every two years), the period is equal to 6 years, the amplitude rise to 4.7. The fluctuations for higher values of ρ are presented in Figure 9d and e. It is worth to notice that for very high values of ρ the cycle is asymmetrical, the 'peeks' are much shorter then the 'depressions'.

Capital productivity growth rate (λ) influences significantly the mode of development, the greater value of the rate λ the longer the fluctuation period although the amplitude of fluctuations are almost the same. For no capital productivity growth (λ=0, i.e. for the 'basic experiment') the fluctuation period is equal to 7.8 years (see Fig. 7) , for 1% growth (λ =0.01) the period increases to 8.5 years, for higher growth rates 2%, 3% and 4% the fluctuation periods are equal 9.8 years, 11.8 years, and 15.2 years, respectively (see Fig. 10). For high values of the growth rate λ we observe saw like fluctuations (see Fig. 10 c and d).

Up to now all simulation experiments have been done for the stable industry (economy). In reality the market is usually a growing one. It turns out the market growth rate γ does not influence the duration of fluctuations but significantly influences the mode of development. For small values of the growth rate γ (less then 3%) the period of fluctuations is not changing but there is superposition of the exponential trend and the observed in the basic experiment 7.8 years fluctuations (see Fig. 11 a and b). For higher values of the growth rate γ the fluctuations are smothered and disappears for very high values (see Fig. 11 c, d and e). Those results are valid for the basic values of the investment delay (τ = 0.8 year) and the price delay (δ=0.5 year) but it turns out that the relationship between delays' values and the growth ratio is much more complicated.

During numerous simulation runs we have noticed that for some values of the growth ratio γ there is a range of the delay values within which the fluctuations are stable (the steady state fluctuations) and are smothered (to reach a stable equilibrium) for values outside of that range. For example for 4% growth of the market (i.e. γ=0.04) the range is equal to (2.2, 51.8), i.e., for τ smaller then 2.2 years and τ greater then 51.8 the fluctuations are smothered (damped) and the economic process is reaching a stable equilibrium, but for τ greater then 2.2 years and smaller then 51.8 we observe a stable (constant) oscillations (although with growing values of the oscillation period, roughly equal to the values presented in Table 1, for different values of delays, but for γ=0; therefore we can say that the values of the fluctuations period do not depend on the growth rate γ). What is interesting the range is the wider the smaller is the growth rate γ. The ranges of the values of τ for different values of γ are presented in Table 2 (see also Fig. 12). As our simulation experiments reveal, for γ =0.02 the fluctuations emerge for the investment delays τ greater then 0.53 (for τ<0.53 the fluctuations are smothered and the process reaches stable equilibrium) but we have been not able to estimate the upper value of the range, therefore we indicated only that it is greater then 100 years. It was also possible to estimate the lower limit of τ for γ= 1.5%, namely equal to 0.25 year. We can expect that for smaller values of γ the lower limit of the delay τ is equal to 0. For values of γ greater then 7.3%, in all simulations we observe quickly damped oscillations and the system is going toward a stable equilibrium.

Conclusions

The hog's cycle model seems to be a good starting point to build the general model of business cycle. It is almost commonly accepted that investment delay is the main factor responsible for emergence of fluctuations in business activity. As our simulation results reveal, the investment delay is really important factor but it is hardly to say that it is the only factor and even not the most important one. Emergence of fluctuations requires a tuning of different factors such as the investment delay (τ), the price delay (δ), sensitivity of producers on signals flowing from the market (*ss*), the growth rate of the market (γ), the capital productivity growth rate (λ), capital depreciation rate (ρ), elasticity of demand (*e*), and probably many others, not included in our simple model (as e.g. credit activity of the banks).

It is seen that all kinds of well known cycles (Kitchin, Juglar and Kuznietz) are observed in the behaviour of the model. The Kitchin 3-5 years cycles emerge for the investment delay of the order 0.3 to 0.6 years and the price delay between 0.2 and 0.3 years (see Table 1). For higher investment delay (one to two years) and the price delay (one to 1.5 years) we observe the Juglar 7-11 years cycles. The Kuznetz infrastructural investment cycles (14-25 years) are observed in our model for the investment delays between 3 to 5 years.

Important emergent property of the model was observed in the series of simulation runs made for the growing market (economy). The growing market acts as a kind of fluctuation filter and the greater the market growth rate γ the more probably is that the fluctuations disappear. For highly growing market (γ >7.3%) there is no fluctuations observed at the steady-state.

We may venture to propose a kind of recipe to avoid (or at least to delimit) economic fluctuations: we ought to create friendly regulatory conditions for entrepreneurs (especially related to entering the market and to proceed business) to ensure highest possible long term growth rate and to minimize investment delays. According to the presented results (see Figure 12 and Table 2) we may expect smooth, non-cyclical, development for 3% growth rate[8] and the delays smaller then 1.2 year (the investment delay might be longer, e.g. 2.2 years but in such a case we ought to ensure the long term growth rate equal to, or greater then, 4%).

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[8] Statistical research suggests that for economically advanced economies (like USA, and GB) long term growth rate in the last 100 years, or so, is around 3%.

Table 1. Period of fluctuations for different values of investment delay (τ) and price delay (δ)

τ / δ	0.0	0.1	0.3	0,4	0.5	0.6	0.8	1.0	1.5	2.0	3.0	4.0	5.0	7.0	10.0
0.0	∞	∞	3.8 (160)	4,4 (90)	5,0 (60)	5.5 (40)	6.4	7.3	9.3	11.2	14.8	18.8	24.3	32.4	44.0
0.1	∞	∞	4.4 (80)	4.9 (50)	5.4	5.8	6.7	7.5	9.4	11.3	14.9	18.9	24.4	32.5	44.0
0.2	3.4 (160)	3.9 (100)	4.9	5,4	5,8	6.3	7.1	7.8	9.7	11.4	15.1	19.3	24.5	32.6	44.1
0.25	3,8 (110)	4.3 (60)	5.2	5.6	6.0	6.4	7.2	7.9	9.8	11.6	15.2	19.3	24.6	32.7	44.2
0.3	4.1 (70)	4.5 (40)	5.4	5.8	6.2	6.5	7.3	8.0	9.9	11.6	15.3	19.4	24.6	32.7	44.2
0.4	4,7	5.1	5.8	6.2	6.5	6.8	7.5	8.3	10.1	11.8	15.3	19.7	24.7	32.8	44.3
0.5	5.3	5.5	6.3	6.5	6.8	7.2	**7.8**	8.5	10.1	11.9	15.4	20.0	24.8	32.8	44.3
1.0	7.2	7,5	8.0	8.3	8.5	8.7	9.2	9.7	11.0	12.4	15.9	21.3	25.4	33.4	44.3
2.0	10.4	10.6	11.1	11.3	11.5	11.8	12.1	12.5	13.6	14.9	17.4	21.4	25.9	35.3	45.4

Table 2. The range of values of investment delays for which the fluctuations are constant (steady state fluctuations)

The market growth rate γ	The range of the delay τ [in years]
0.015	0.25 to ‘more then 100 years’
0.02	0.53 to ‘more then 100 years’
0.03	1.2 to 75.9
0.04	2.2 to 51.8
0.05	3.3 to 35.2
0.06	4.5 to 22.9
0.07	6.6 to 13.9
0.072	7.5 to 12.2
0.073	8.2 to 11.0
greater then 0.073	no fluctuations

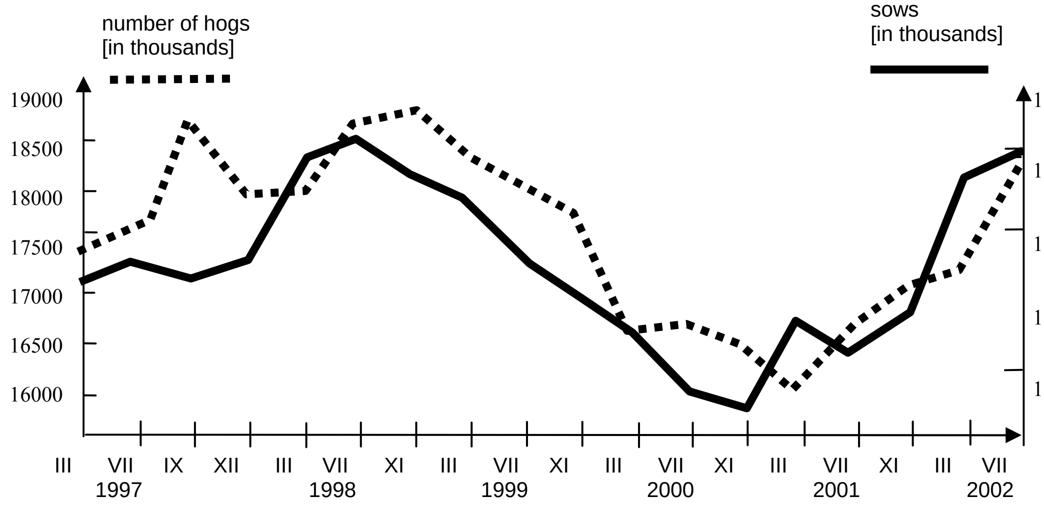


Figure 1. Hogs’ stock in Poland (after Gazeta Wyborcza, *12 October 2002)*

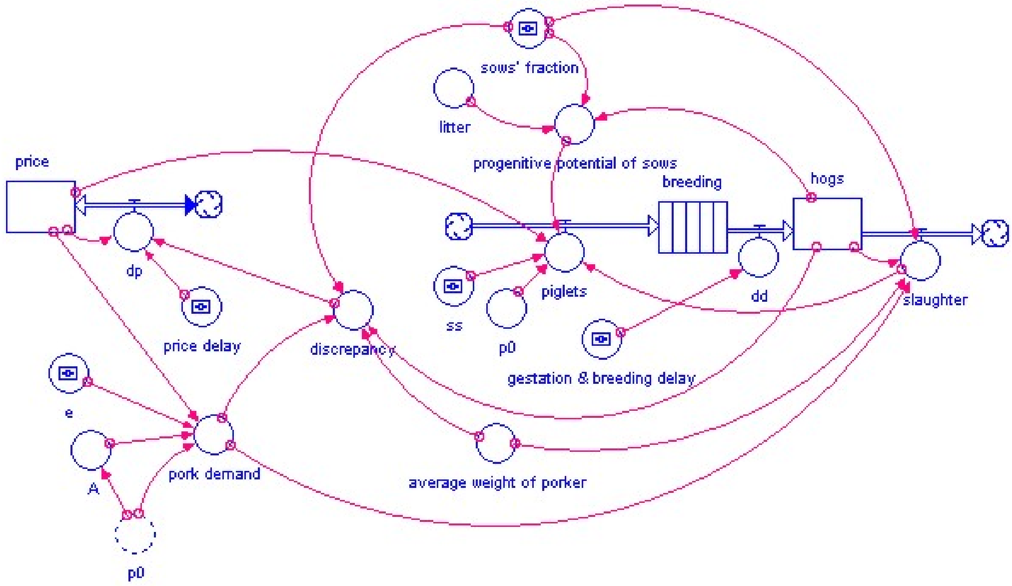


Figure 2. Hog's cycle model

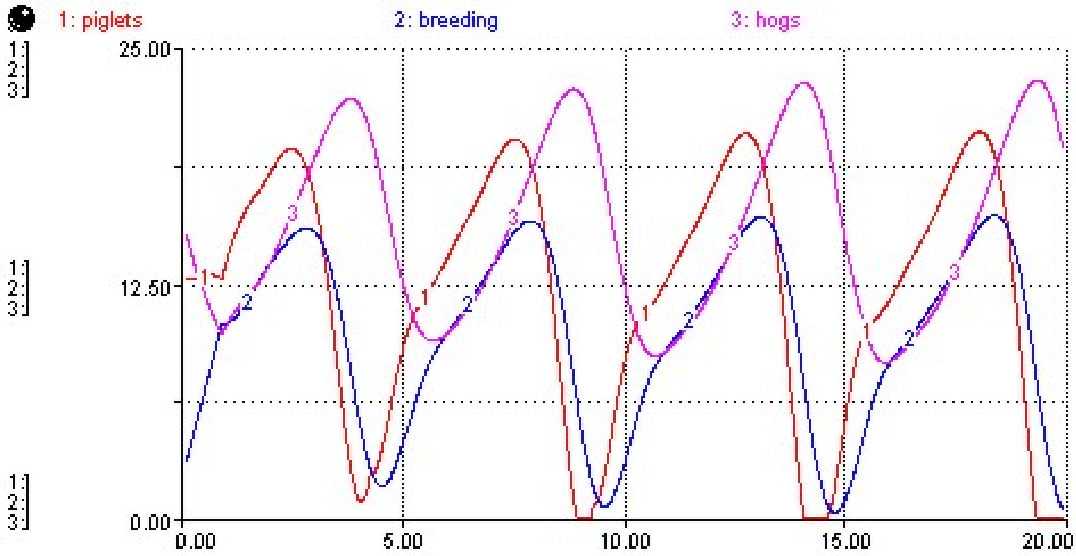


Figure 3. Fluctuations of number of piglets, hogs in breeding process and hogs

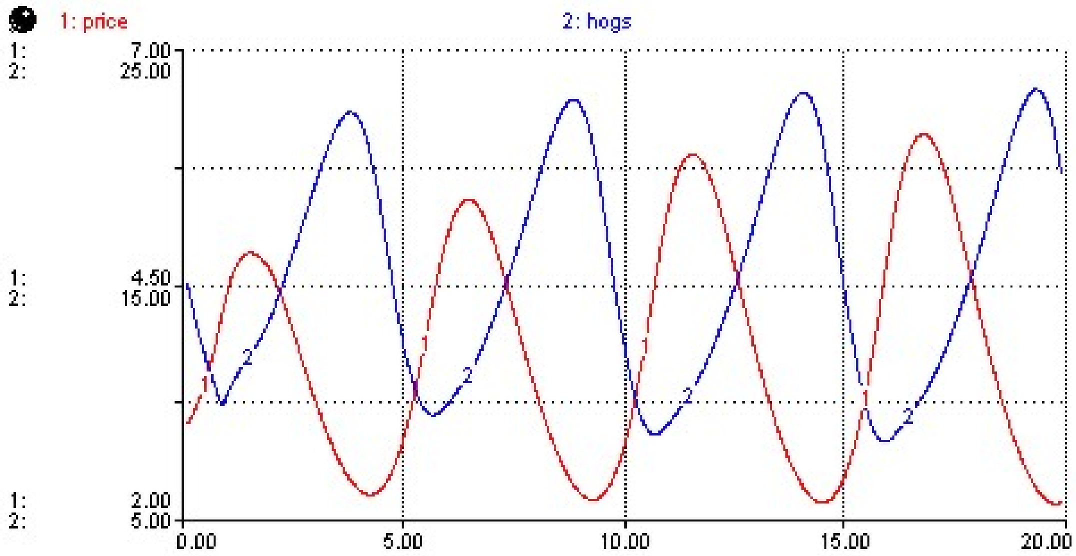


Figure 4. Counterpace fluctuations of the pork price and the number of hogs in the hog's cycle model

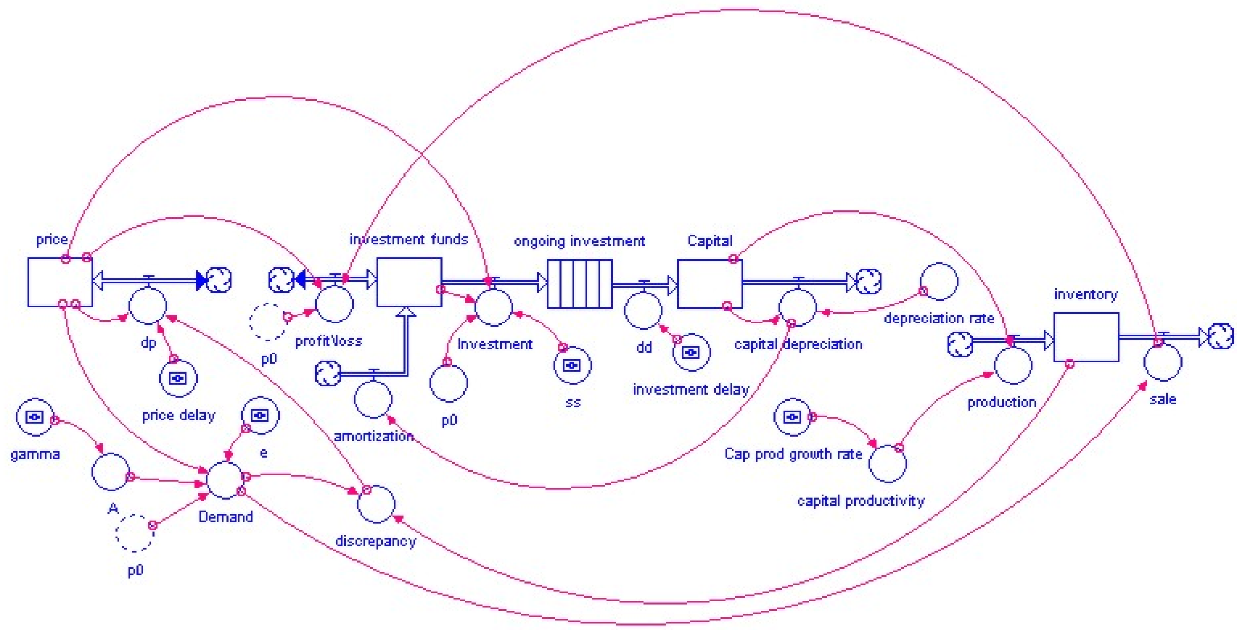


Figure 5. The flow diagram of the business cycle model

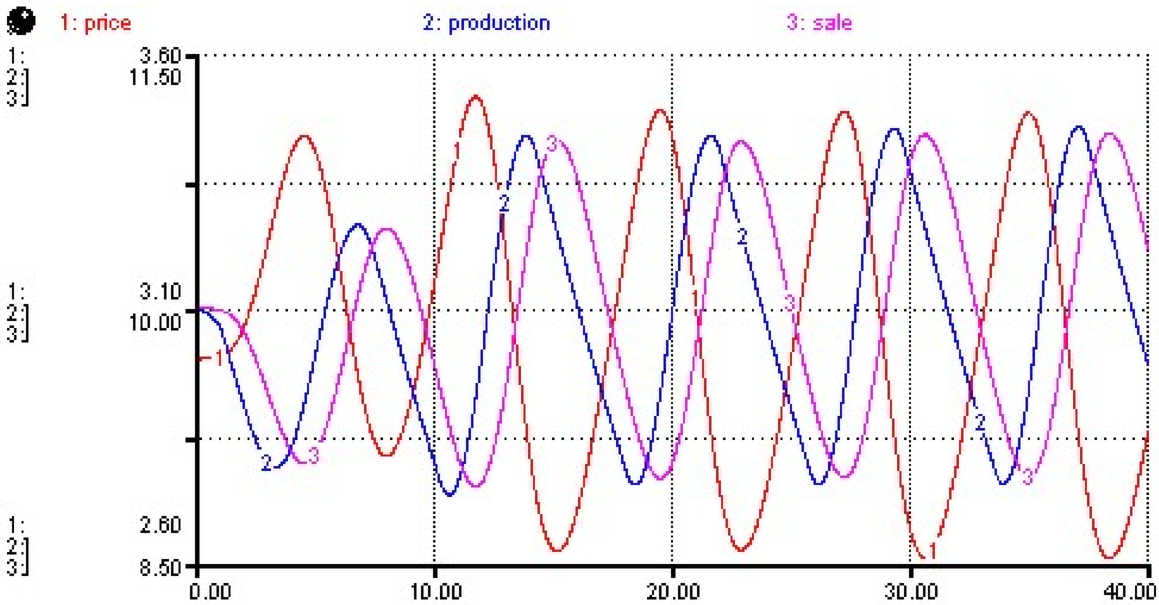
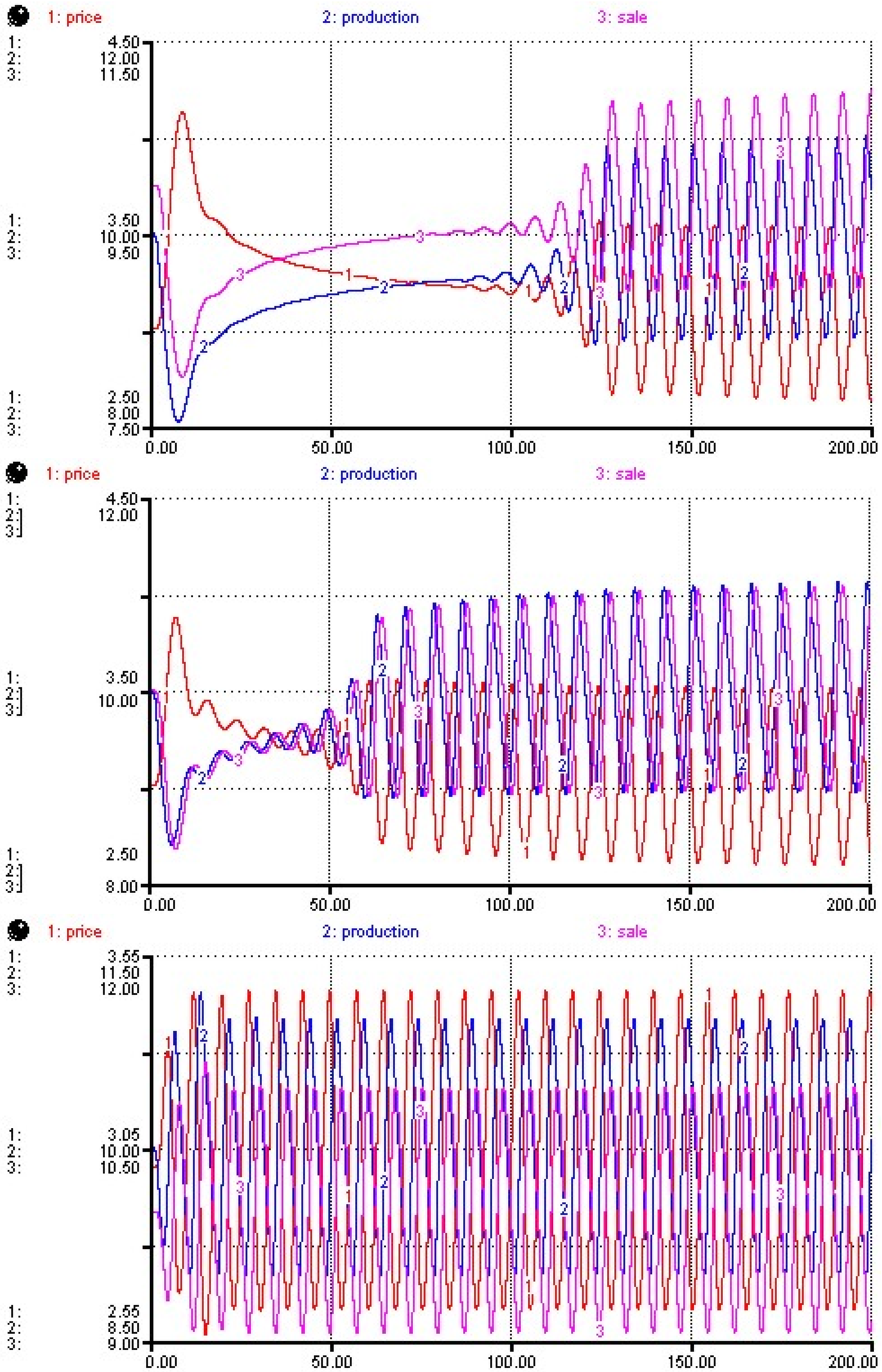


Figure 6. Price production and sale for basic values of the model's parameters



a)

ss=0,2

b)

ss=0,4

c)

ss=7,0

Figure 7. Modes of development for different values of sensitivity parameter ss

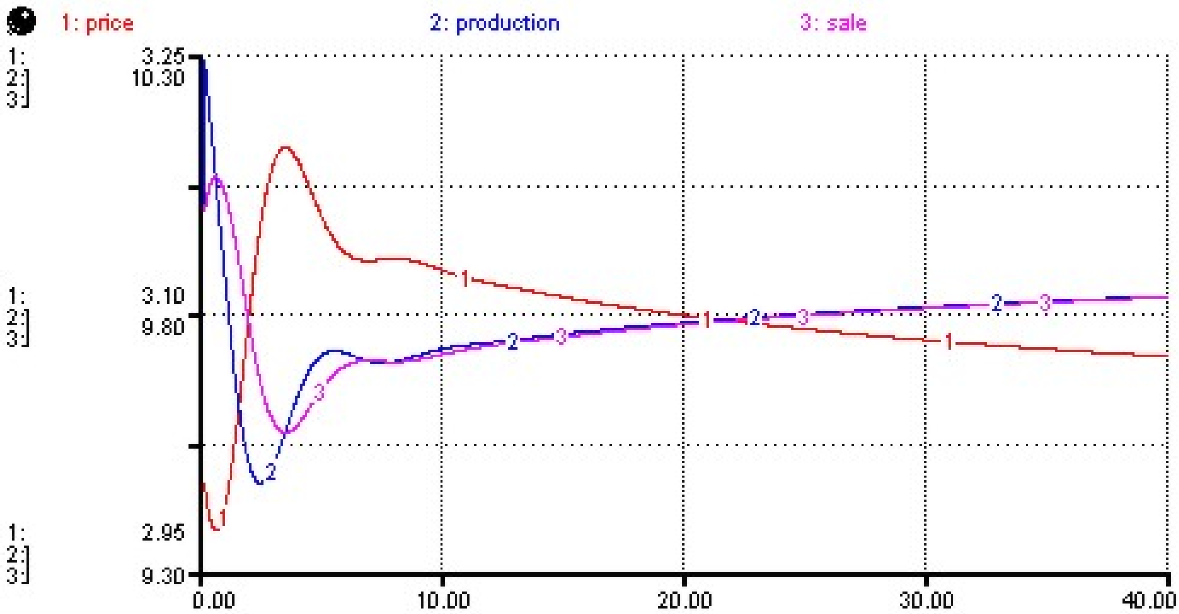
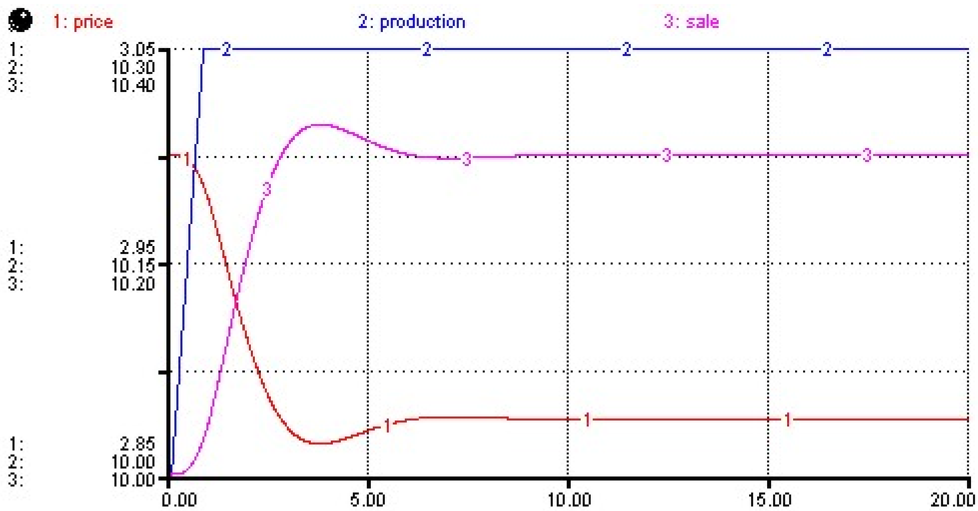
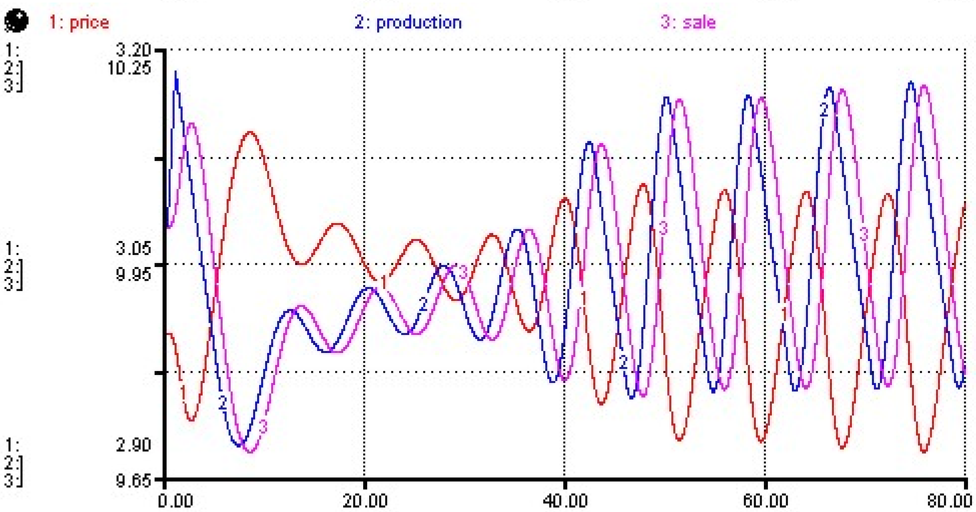


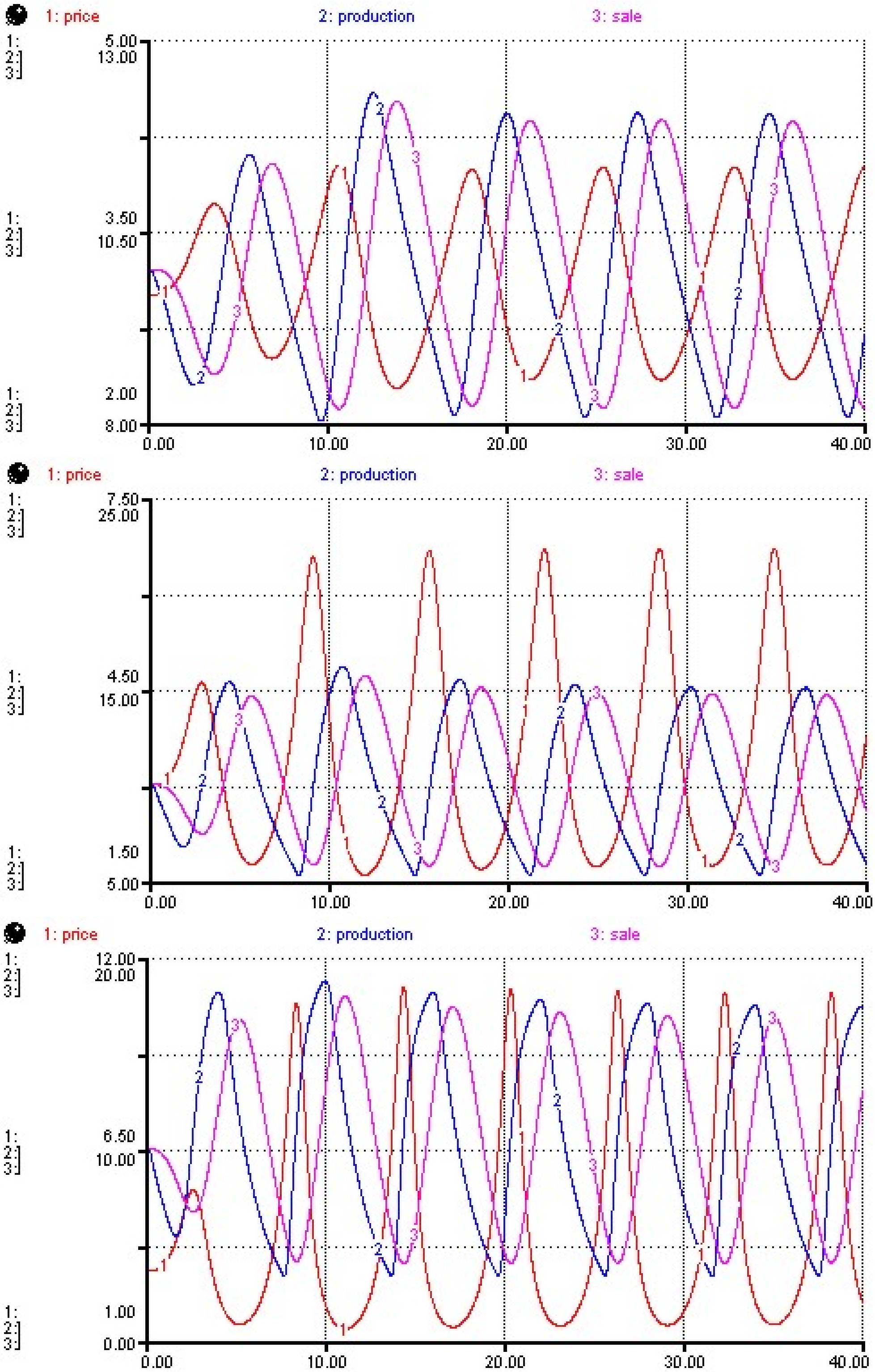
Figure 8. No price and investment delays



a)
ρ =0.0



b)
ρ =0.01

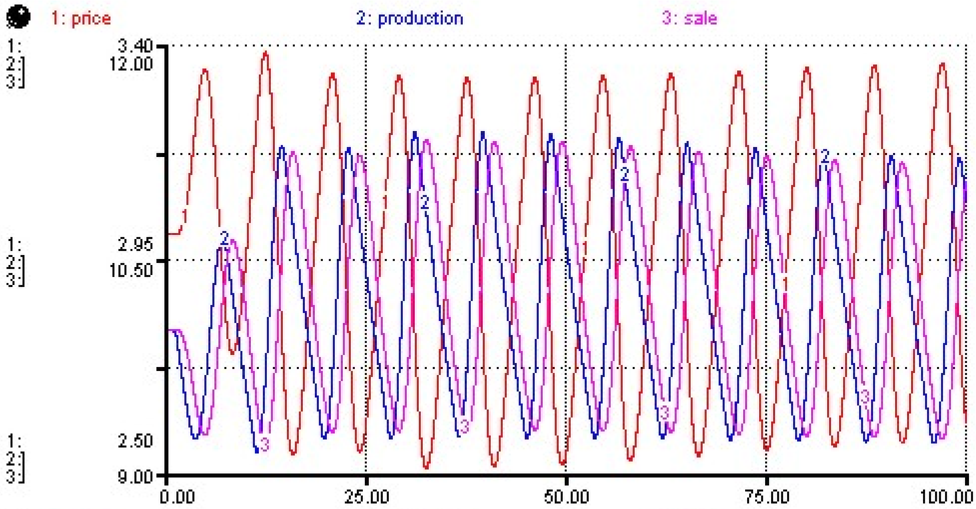


c)
ρ =0.1

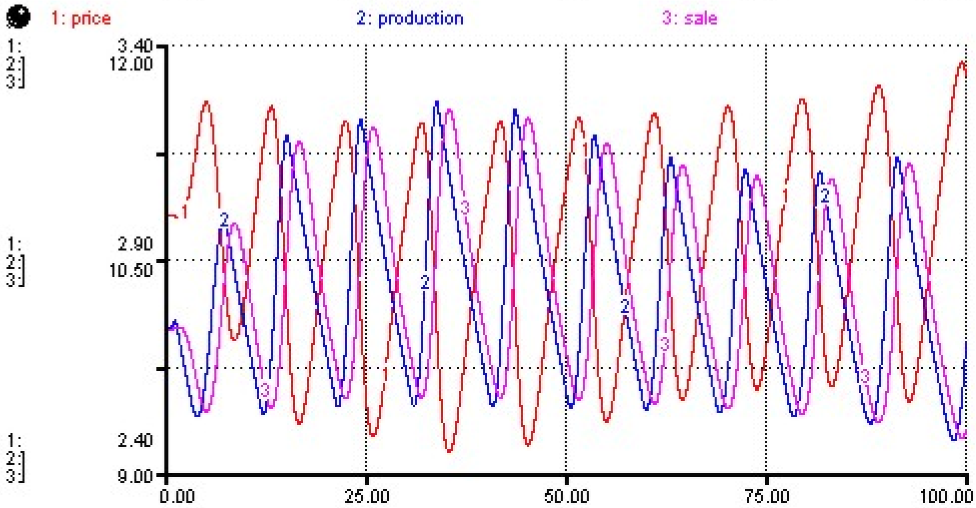
d)
ρ =0.3

e)
ρ =0.5

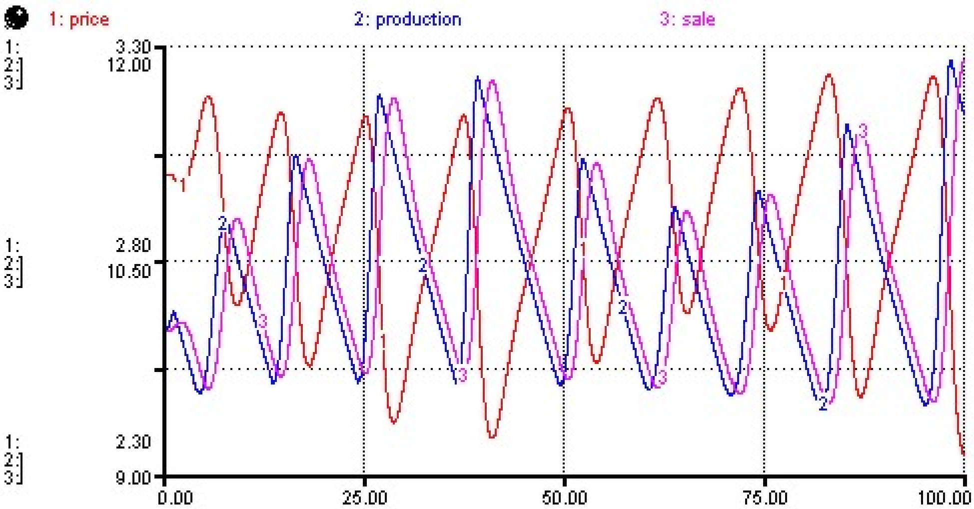
Figure 9.Fluctuations for different values of the capital depreciation rate ρ



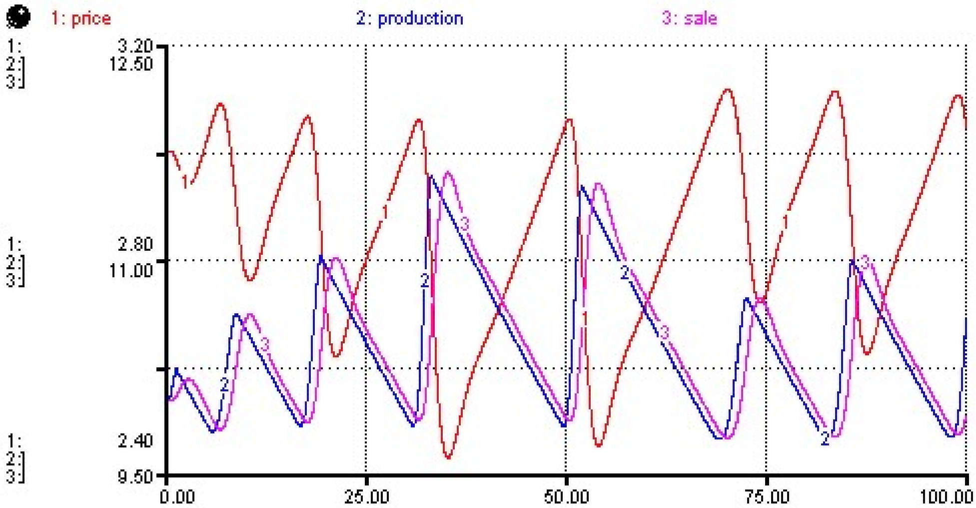
a)
λ=0.01



b)
λ=0.02

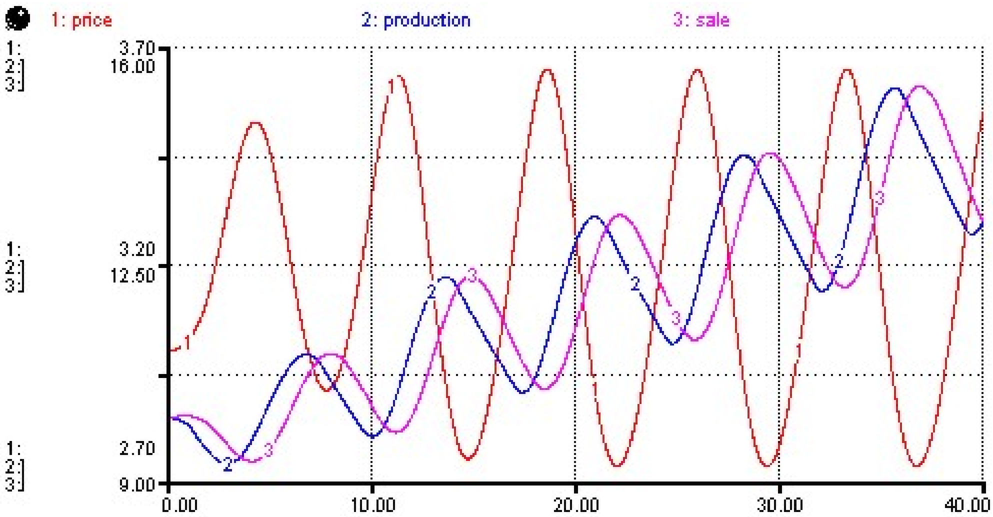


c)
λ =0,03

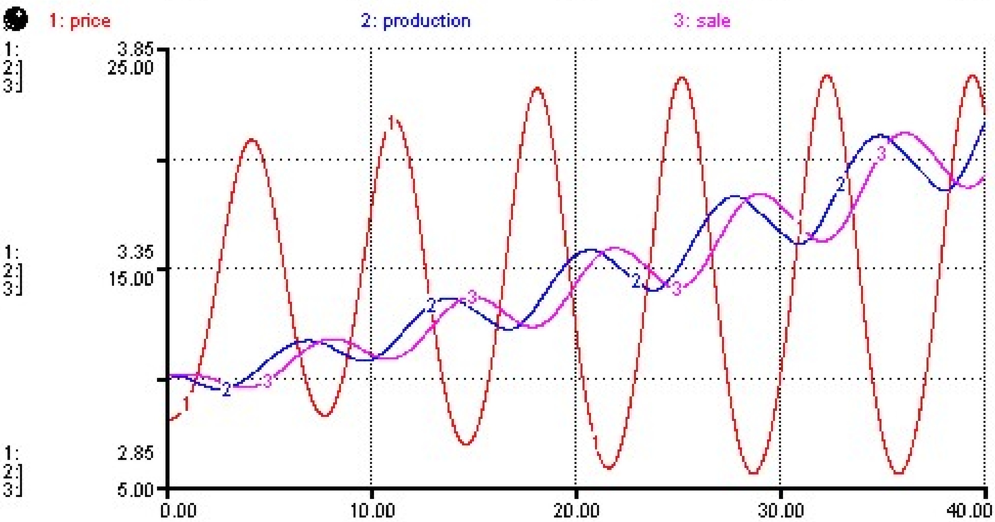


d)
λ =0.04

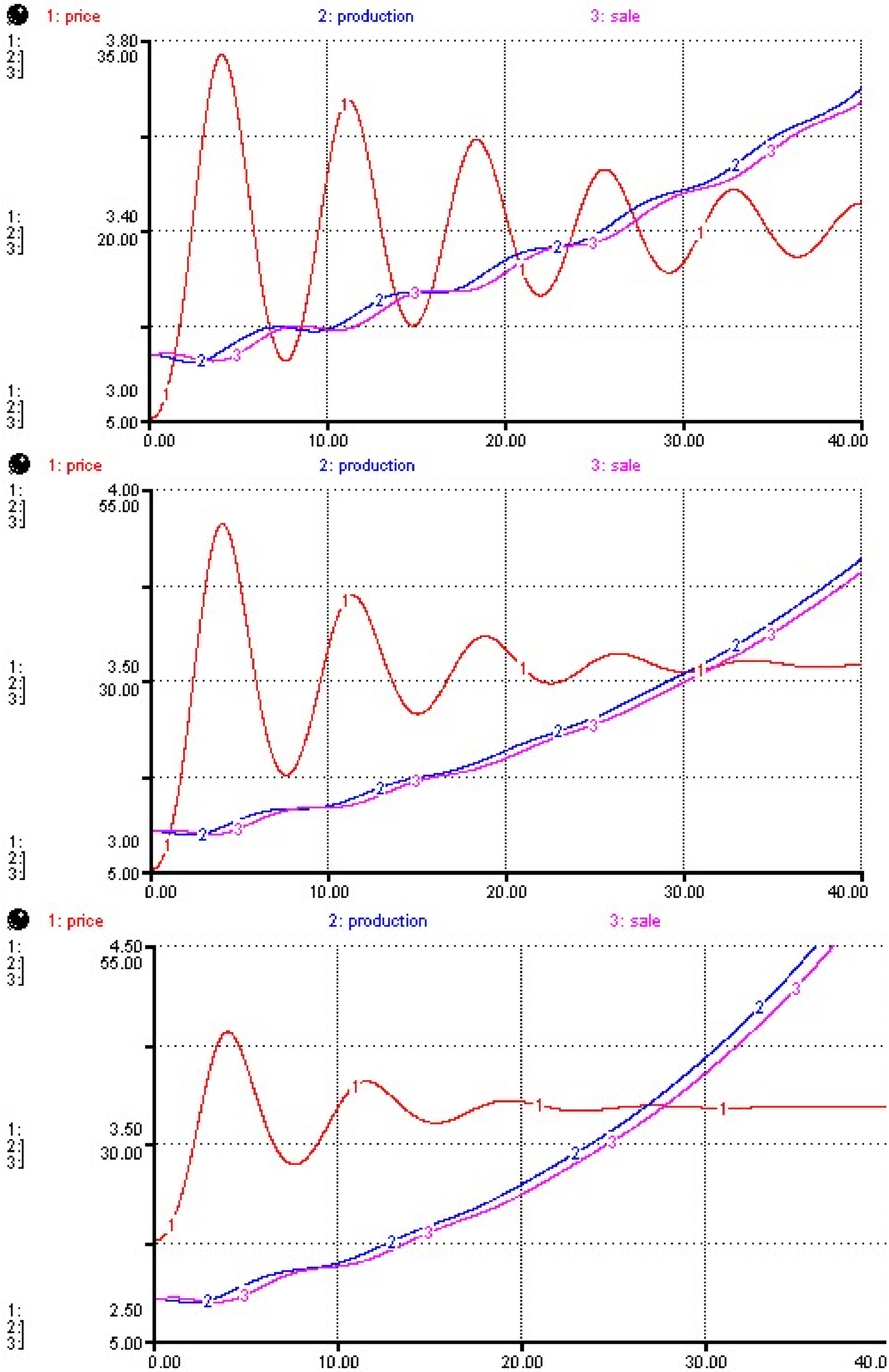
Figure 10. Capital productivity of rate



a)
γ=0,01



b)
γ=0,02



c) γ=0.03

d) γ=0.04

e) γ=0.05

Figure 11. Market growth rate and fluctuations

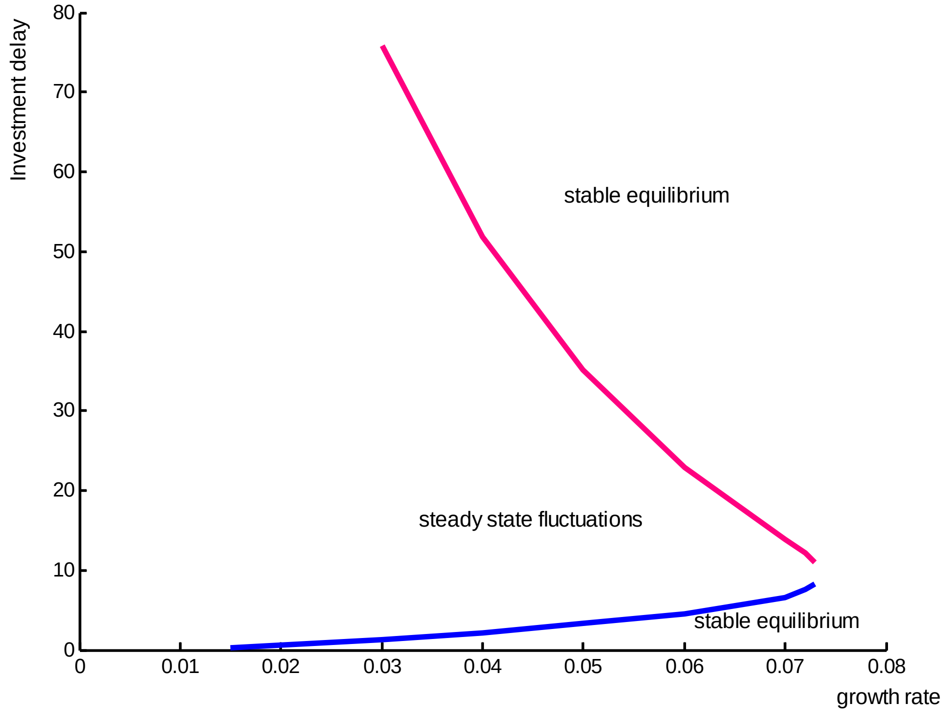


Figure 12. The investment delay (τ), the growth rate (γ) and the steady state fluctuations