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SIMULATION OF SOME PROCESSES OF DEVELOPMENT

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We consider successive generations of elements performing the some function in specific environment. We assume that elements are differently prefered with regard to their types, are reproduced with respect to these preferences and that sometimes the types of elements are modified during reproduction. We give the appriopriate stochastic model describing the variations of the generations state resulting from such reproduction. We are of opinion that properties of this model fit well to those observed in some real life processes of development of evolutionary type. Several simulation examples are presented to uphold this thesis.

1. INTRODUCTION

We consider a population of elements of several types acting in some environment. All elements do essentially the same job - meet the same environment requirment, though they can accomplish it differently and be differently prefered by environment. We assume the specific environment in which quality index assigned to element and reflecting this preference is random variable with probability distribution the same for all elements of the same kind, acting in the same period of time. We will call the environment the more homogenous the less is a variance of quality index. The environment has a limited capacity. Elements are introduced to environment, act there some time and are removed. The difference between current capacity and number of elements acting forms a demand for new elements.

We assume a discret time and consider a process illustrated on fig. 1. In each period there exists a population which occupy the environment. It consists of that elements which "survived" from previous population and of new generation of elements supplied by reproduction process. The reproduction is a random process with tendency to meet the environment demand and to increase the number of elements with high quality on expense of elements with low quality. The elements of particular types are reproduced seperately and the variance of reproduction ratio diminishes with growing number of elements reproduced. There exists a peak ratio of reproduction for each type which can not be exceeded and as a result-the upper limit of elements number in next generation determining the extent to which the demand of environment can be fulfilled. When the peak ratios of reproduction are not involved, elements are reproduced in such a fashion that the expected number of all new elements equals the demand and expected value of reproduction ratio for particular type is proportional to its quality. There exists slight probability that element can be modified during reproduction into another type, sometimes with better quality, then those hitherto represented in the population. As a result of such a reproduction, generation state, defined as elements distribution between particular types, changes in time, what is follo-

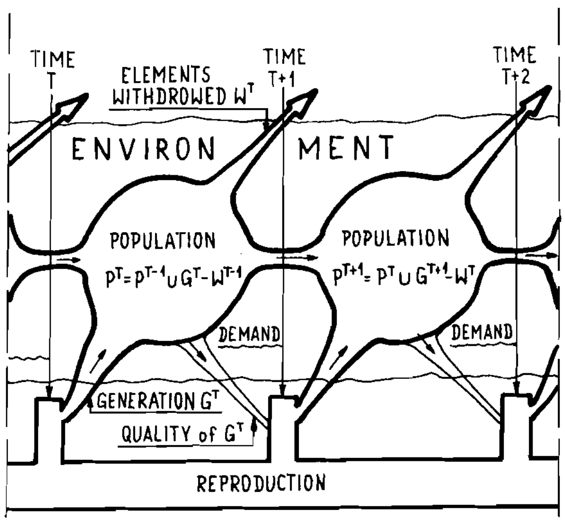

Fig. 1. Elements flow in successive populations.

wed by adequate changes in the population state.

In next section we propose a matematical model describing the process of variation of the generation state in succesive time periods. Using Waddington terms [1] it describes the behaviour of progressive system having the homeorhesis property. We intend to point that discussed process exhibits a far going analogy with observed, real life development processes. In particular, there occurs the steady growth of the average generation quality, combined with cyclic changes of the generation state, as a successive types of higher and higher quality are introduced and gain domination in it.

There is quite a lot of papers, mainly biologically orientated, which deals with mathematical models of similiary defined populations. It begins probably with Fisher [2], of more recent these of Eigen [3], Karlin [4], Tuffner-Denker [5] should be mentioned in this context. The main innovation introduced in our model seems to be a vector space of types with quality function defined on it. It allowed the modification process to be adequately included into model.

One can meet remarks, e.g. [6] that other then biological development processes of evolutionary types can be considered in similar fashion. We tried to use neutral terms, and perhaps the reader can share our opinion that above description reflect to some extent, e.g. a process of competition on specific market.

2. MODEL

Let the type of element be determined by n parameters: $d_1,\dots,d_n$ assuming integer values. Then each type d^i can be represented as a point: $(d_1^i,\dots,d_n^i)$ in n-dimensional discrete parameters space D^n:

$$D^n=\{d\}\subset R^n:\ x\in D^n \Leftrightarrow x\in R^n \wedge x_i=\mathrm{Entier}(x_i),\ i=1,\dots n. \quad (1)$$

Let the norm:

$$\|d^i-d^j\|=\sum_{k=1}^{n}|d_k^i-d_k^j| \quad (2)$$

gives a distance between types i and j. Let T = 0,1,2,... denotes discrete time=generation number. Let G^T denote the T-th generation of elements. The state of generation is given by a distribution function:

$$N:\ D^n X T \longrightarrow D_+ \quad (3)$$

where $D_+=\{0,1,2,\dots\}$

$N(d^i,T)=N_i^T$ determines the number of i-type elements in G^T. As D^n is the denumerable set of points, all types d^i can be numered successively: d^1, d^2, d^3,... and the generation state can be denoted by the vector:

$$S^T=\left[N_1^T,\ N_2^T,\ N_3^T,\dots\right]^{tr} \quad (4)$$

We assume there are given in advance:

- Quality function:

$$q:\ R^n X T \longrightarrow R_+ \quad (5)$$

$q(x,T)=q_i^T$ for $x=d^i$ determine expected quality of i-type elements in G^T.

- Maximum reproduction function:

$$r:\ D^n X T \longrightarrow R_+ \quad (6)$$

$r(d^i,T)=r_{mi}^T$ determines the peak reproduction ratio of i-type elements in G^T.

- Modification function:

$$m:\ D^n X D^n X T \longrightarrow R_+ \quad (7)$$

$m(d^i,\ d^j,T)=m_{ij}^T$ determines the probability of modification from i to j-type during reproduction of G^T, $\sum_j m_{ij}=1$.

- Demand function:

$$M:\ T \longrightarrow D_+ \quad (8)$$

$M(T)=M^T$ determines the demand of environment for new elements in period T.

- Probability distribution of quality evaluation:

$$f(\tilde{q},q_i^T),\ \tilde{q}\in R_+,\ E[\xi_i^T]=q_i^T \quad (9)$$

where ξ_i^T is a random variable with distribution f determining the quality index assigned to i-type elements in G^T.

- Probability distribution of reproduction:

$$g(\tilde{N},E[N_i^{T+1}]),\tilde{N}\in D_+,\ E[\eta_i^T]=E[N_i^{T+1}] \quad (10)$$

where η_i^T is a random variable with distribution g determining the number of "progeny" of i-type elements in G^T.

To evaluate the state of next generation we determine successively:

1. Average quality of i-type:

$$\bar{q}_i^T=\frac{1}{N_i^T}\sum_k \tilde{q}_{i,k}^T,\quad \text{for all } i \quad (11)$$

where $\tilde{q}_{i,k}^T$ is a realization of random variable for k-th element of i-type, eq. (9).

2. Total reproduction ratio of i-type:

$$r_i^T=\min(a^T\cdot\bar{q}_i^T,\ r_{mi}),\ \text{for all } i \quad (12)$$

where $a^T\in R_+$ is a factor common for all types adjusting reproduction to the demand:

$$a^T=\min_a\left(\left|M^{T+1}-\sum_i N_i^T r_i^T(a)\right|\right) \quad (13)$$

3. Reproduction ratio from i to j-type

$$r^T = r_i^T m_{ij}^T \qquad \text{for all } i,j \qquad (14)$$

4. Expected state of T+1 generation:

$$E[S^{T+1}] = [r_{ij}]\ S^T \qquad (15)$$

where r_{ij}^T is an infinite-dimensional matrix of reproductions ratios.

5. State of T+1 generation:

$$S^{T+1} = [\tilde{N}_1^{T+1}, \tilde{N}_2^{T+1}, \tilde{N}_3^{T+1}, \dots]^{tr} \qquad (16)$$

where $\tilde{N}_i^{T+1}$ is a realization of random "progeny" variable η_i^T eq. (10).

The above model describes to some extent the process discussed in introduction. Simulation results, presented further on, have been obtained under some simplifying assumptions which are as follows:

- We generated random values for the reproduction only, using the Poisson Distribution to this purpose - then the described process can be considered as a multitype branching process[7]. As the matter of fact, there is no need to generate random values for those processes seperately, as one complex distribution can be educed and used for simulation.

- We assumed the constant demand M and the constant peak reproduction ratio r_m the same for all types. In such situation we could simplify the determining of reproduction ratio using instead eqs. (11) and (12) the formulae:

$$r_i^T = \left[1 + (r_m - 1)\ \frac{M - N_0^T}{M}\right] q_i^T / q_0^T \qquad (17)$$

where N_0^T is a total number of elements in G^T and q_0^T is an average quality of G^T:

$$q_0^T = \frac{1}{N_0^T} \sum_i N_i^T q_i^T \qquad (18)$$

It can be shown that if $r_i^T = q_i^T / q_0^T$ the expected number of all elements G^T is the same for all generations - the Critical Galton-Watson Process, If N_0^T does not excede M considerable and $1 < r_m < 2$, what just occured, remaining factor acts to reduce a difference between M and N_0^T.

- We assumed the modification probability m_{ij} depends only on distance between i and j-type and is given as follows:

$$m_{ij} = \begin{cases} 0 & \text{if } \|d^i - d^j\| > 2 \\ (m_0)^2 & \text{if } \|d^i - d^j\| = 2 \\ m_0 & \text{if } \|d^i - d^j\| = 1 \\ 1 - \sum\limits_{j, j \neq i} m_{ij} & \text{if } \|d^i - d^j\| = 0 \end{cases} \qquad (19)$$

where $m_0 \ll 1$.

The last thing to discuss here is a choice of quality function, which is a matter of importance for appriopriate interpretation of simulation results. We chose a shape roughly analogous to this of an undercut, narrow ridge gradually elevating in direction not parallel to the axies. It is a form similar to the Waddington's chreods [1], the only difference being - we deal with maximization not minimization. There is similar motivation for this choice also. We can often describe the development process of some objects indicating parameters achieved in successive generations and resulting trajectory correlated with quality growth. Any time, any serious deviation from this trajectory gives a rapid drop of quality. A better solution is not easy to be found and generaly requires a change of several parameters simultanously, as parameters are usualy well tuned up. The chosen metric of D^n reflects this property as it "rates higher" the change involving more parameters.

3. SIMULATION

3.1. Typical features of simulated processes.

The dynamics of the generation state is a result of two trends composed. The first trend arising from selective reproduction, tends to increase the number of elements of types with quality q_i higher then average quality q_0^T, and to reduce the number of others. This tendency is the stronger, the greater (the lesser) is a ratio q_i / q_0^T. As an average quality increases during such process, this trend acts to supersede all elements by temporarily the best ones - to attract the generation distribution to the point representing the timely best type. The second trend, arising from modyfications gives elements of types deviated from this being reproduced - it acts to disperse the generation distribution around. This two trends taken together can give a state of quasi-equilibrium. In the parameters space, there exists a center represented by the type temporarily the best containing a majority of elements and certain surrounding of this center, composed of types with worse quality but because of modifications still containing some elements. When due to modifications the elements of new better type are introduced to generation, the center of distribution moves to the point representing this new type.
To illustrate the typical features of generation dynamics we use data obtained from simulation carried out under following assumptions:

$M = N^0 = 10^4$, $m_0 = 1.5 \cdot 10^{-3}$, $n = 2$

$N^0(15,15) = 10^4$ and zero elsewhere

$$q(d_1, d_2) = \exp\left\{-10^{-2}\left[(d_1 + d_2)^2 + (d_2 - d_1)^2\right]\right\} \qquad (20)$$

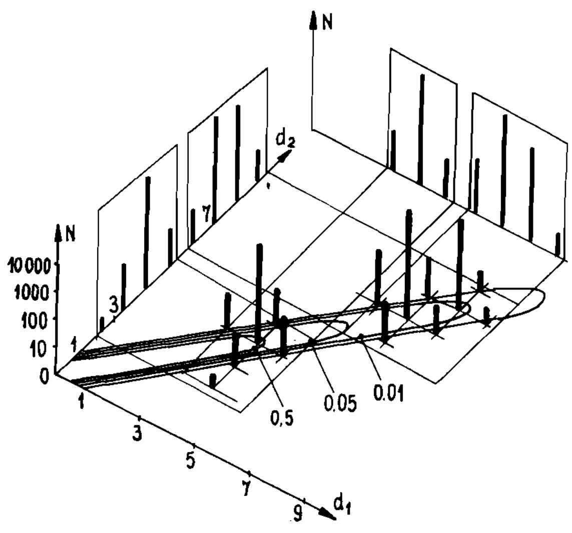

Fig.2. Parameter space, quality function and G^{110} and G^{250} states.

The state of 110-th and 250-th generations is shown on fig. 2. The generations are so distant, that they are composed of elements of entirely distinct types. The 110-th generation is in transition stage, the center of distribution just moves from point (8,8) to point (7,7). The 250-th generation is in quasi equilibrium stage. Most of elements is concentrated in the point, (4,4) the better type has not yet been found. On the axies, the distributions of parameters for each generation have been indicated. The first moments of these distributions are the coordinates of generation center. Note, that logaritmic scale for the number of elements has been applied.

On the same figure the contour lines of the quality function are indicated. As can be seen, the function has such a shape that a change of single parameter of dominating type gives a type with worse quality. To get the better type it is necessary to modify two parameters in proper direction. In quasi-equilibrium state practically all elements are of dominating type and this type quality is the average quality of generation. As the assumed probability of double modification is $(1.5)^2 \cdot 10^{-6}$, there are two ways to get it and the number of elements is 10^4, the successful double modification can be expected one a 45 generations. In a surrounding of dominating type there are two others which can give a better type by a single modification. If their quality is 0.8 of the average, then in equilibrium state both should be represented by 150 elements - each modification brings 30 element and each reproduction takes of 30 elements. In such a group of elements the successful modification can be expected one a 4 generations. From comparison of this number with that obtained for double modifications follows that the main source of successful modifications is not the dominating type but the types belonging to its surrounding.

The "history" of generation is drawn on fig. 3. Successively types (7,7), (6,6), (5,5), (4,4) gains and lose domination in the generation. Most of the time there is only one type dominating. All the time, exists a "background" consisting of elements of types from surrounding of generation center. They have usualy worse quality then those which dominate. Their number is a small fraction of total number of elements present in generation.

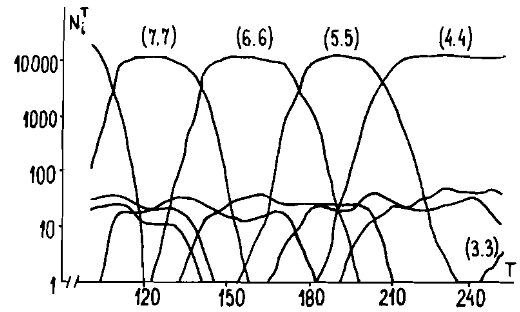

Fig.3. Elements of particular types in successive generations.

The average quality obtained in successive generations is drawn on fig. 4.

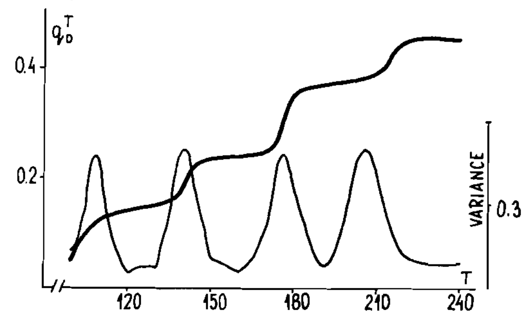

Fig.4. Average quality and variance in successive generations.

It can be seen that periods of rapid growth of quality are correlated with transition stages of generation. On the same figure the variance of generation is traced. The lowest level of variance is observed in quasi-equilibrium state. The simulation results presented in this paper are for deterministic quality evaluation and for random reproduction with Poisson Distribution. The question can arise, to what extent the random factors impact the model behaviour. It appears there exists a strong deterministic component, especialy when more numerous types or the best types are involved. Fig. 5 gives for successive generations the number of elements of two distinct types obtained in four different realizations. The results are for: $M = 10^3$, $m_0 = 0$. Initial state:

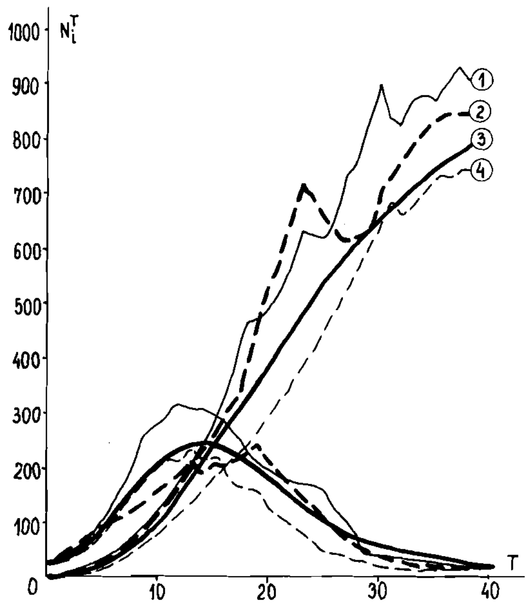

Fig. 5. Elements number in realizations assuming different evaluation-reproduction procedures.

$N_1^0=40$, $N_2^0=30$, $N_3^0=20$, $N_4^0=8$, $N_5^0=2$,

$\bar{q}_1^0=1.0$, $\bar{q}_2^0=1.1$, $\bar{q}_3^0=1.2$, $\bar{q}_4^0=1.3$, $\bar{q}_5^0=1.4$

For each of those realizations another evaluation-reproduction procedure has been used:

1 random evaluation with uniform distribution - the marigins of distribution couple together and deterministic reproduction:

$N_i^{T+1} = \text{Entier}\left(E(N_i^{T+1})\right)$,

2 deterministic evaluation and random reproduction with:

$q_i^T = q_i^T + 0.15\sin\left(\frac{\pi}{5}T + \varphi_i\right)$,

3 deterministic evaluation and deterministic reproduction,

4 deterministic evaluation and random reproduction with Poisson Distribution.

Numbers of elements for 3-rd and 5-th types are drawn.
Irrespective of small size of generation - 100 elements in initial state - what increase the impact of random factors, realizations seems to be quite well tuned.

3.2. Impact of parameters on development rate

We consider the development rate as given by a slope of the curve representing the average quality of successive generation. To illustrate the influence of particular parameters of the model discussed on the development rate, we use data obtained from simulation, carried out with following quality function:

$$q(d_1,d_2) = \left(1-2\cdot10^{-3}\left((d_1+d_2)^2 + \frac{a_s}{a_i}(d_2-d_1)^2\right)\right)^4 \quad (21)$$

where: $a=\sqrt{a_s/a_i}$ - asymetry coefficient giving a ratio of ellipse axies.
As this function is less steep then that given by eq. (20) the stages of transition and quasi-equivalence are less distinct.

Fig. 6. gives the average quality obtained for generation size N_0^0 equal: 10^4, 10^5, 10^6. The results are for: $M=N_0^0$, $m_0=0.01$, $a=4$, $S^0:N(30,30)=N_0^0$ and zero elsewhere.

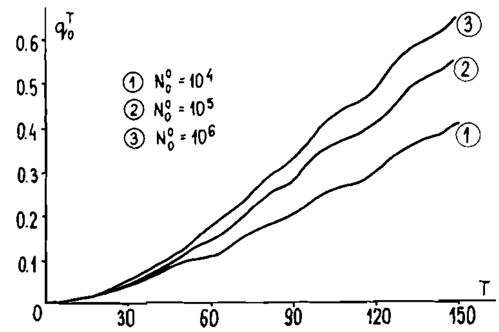

Fig. 6. Impact of generation size on average quality.

The more numerous the generation is, the more rapid development is obtained.

Fig. 7. gives the average quality obtained for probability of modifications m equal: 10^{-1}, 10^{-2}, 10^{-3}. The results are for: $M=N^0=10^4$, $a=4$, S^0: $N^0(30,30)=10^4$ and zero elsewhere. The greater is the probability of modifications the more rapid development is obtained.

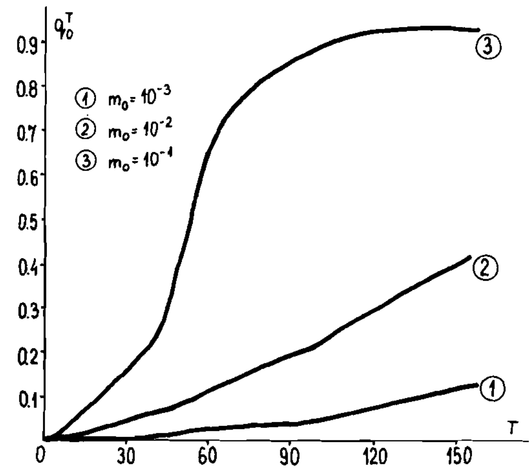

Fig. 7. Impact of probability of modification on average quality

For $m_0=10^{-1}$, after 120 generations dominates the best type possible: d(0,0) - steady state. Due to high rate of still occurring modifications the q_0^T is only 0.94 of maximal quality q(0,0)=1. This cut in quality can be considered as a price of rapid development.

Fig. 8. gives the average qualities obtained for different asymetry coefficients a equal: 1, 4, 16. Results are for: $M=N^0=10^4$, $m_0=10^{-2}$, S^0: $N^0(30,30)=10^4$ and zero elsewhere. The greater is the asymetry of axies the more difficult the development is.

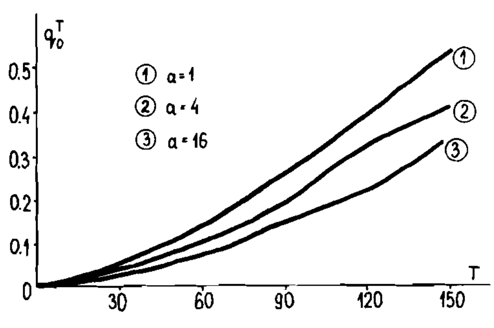

Fig. 8. Impact of asymetry of quality function on average quality.

Fig. 9. gives the map of quality functions with trajectories of the generation center for different starting points drawn in. We assumed that whole population is initially of one type respectivelly: d(11,11), d(13,7), d(15,4), d(15,0). The results are for: $M=N^0=10^4$, a=4.

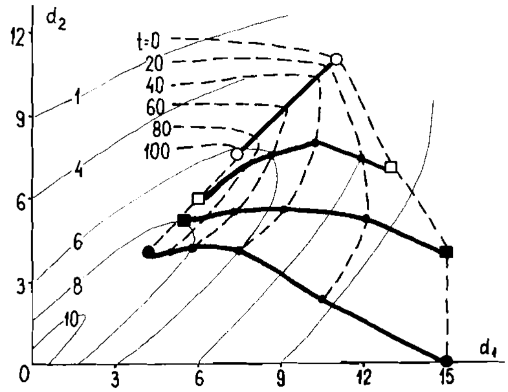

Fig. 9. Generation center trajectories for different starting points.

Fig. 10 gives average qualities obtained from different starting points. It appears that the lowest rate of development is obtained when the center of generation proceeds along the ridge of quality function the same time trajectories tends to proceed along this ridge . As the shape of quality function reflects environmental preferences, the situation when generation finds itself outside the ridge can be interpreted as a result of a rapid change of environment.

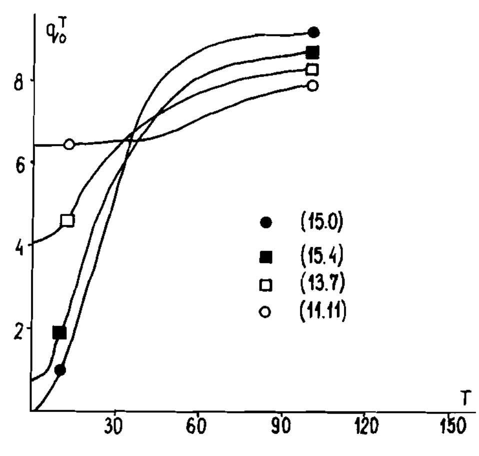

Fig. 10. Impact of initial state on average quality.

3.3. Some special cases

Figs. 11 and 12. illustrate the generation behaviour when the quality function has a local maximum. The results are for different m_0 equal: 10^{-1}, 10^{-2}, 10^{-3}, and $M=N^0=10^4$, S^0: $N^0(12,12)=10^4$ and zero elsewhere. Quality function is given on fig. 11. The trajectory of generation for $m_0=10^{-2}$ has been drawn, • marks the center position for every twenty generations.

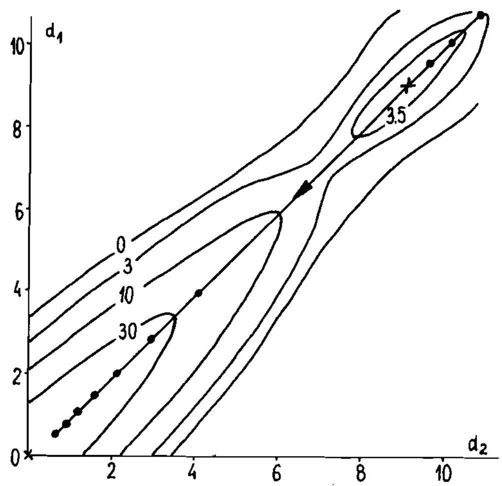

Fig. 11. Passage through local maximum.

Fig. 12. illustrate the ratio of development with respect to m_0. The higer the modification probability is, the easier the generation pass through the local maximum. The price of more rapid development is a lower quality in stationary state - compare curves 2 and 3.

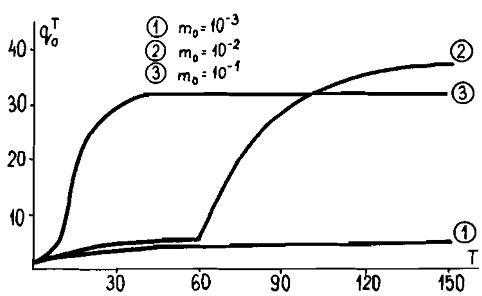

Fig. 12. Passage through local maximum, probability of modification influence on average quality.

Fig. 13. illustrate the situation when there are two possible ways of development. As the slope of both ridges nearby bifurcation point is similar, the choice of further way of development is quite random. Once the generation enter one ridge, the probability it can change its way diminishes rapidly. The probability of proper choise increases with m_0. For: $M=N^0=10^4$, $m_0=10^{-2}$, S^0: $N^0(13,9)=10^4$ four realizations have been computed, 3 on 4 "gone" towards the lower maximum. When $m_0=$ $= 4\cdot 10^{-2}$ has been assumed the realization "gone" towards the higher maximum.

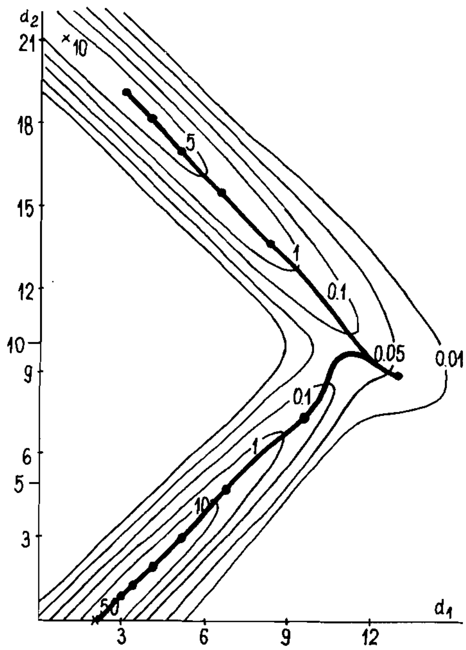

Fig. 13. Passage through bifurcation point.

It is obvious that an assumption that probability of modification depends only on distance batween elements is over simplified with respect to real life processes of development. Promising modification should be made "outside" of existing types and especialy "at the forefront" of developing generation. It seems also in some sense easier to get a product uniting two existing features then a product with quite new feature - parameter value.

Fig. 14. allows to compare results obtained for "normal" process with that obtained under assumption that every time when it is possible to get a new type by recombination of parameters of existing types, elements of this combined types are introduced to generation. The results are for: $M=N^0=10^5$, $m_0=10^{-3}$, S^0: $N(30,30)=10^5$, $q(d_1,d_2)$ is given by eq. (21). The rapid growth of development rate is clearly visible.

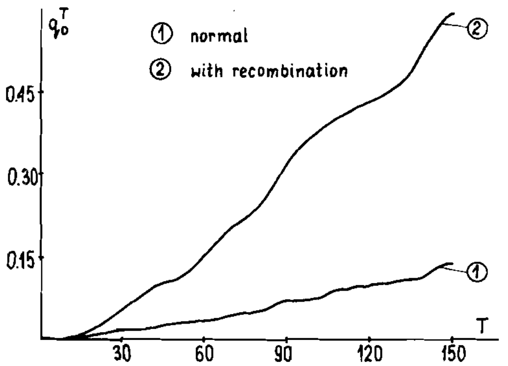

Fig. 14. Impact of recombinations on average quality

The last case to be shown here is case of "pursuit". We compare two generations on different level of development, developing separately along the some ridge. The curves 1 and 4 on fig. 15 show the average quality obtained by elements of this two generation. Results are for: $M=N^0=10^4$, $m_0=1.5\cdot10^{-3}$, $q(d_1,d_2)$ is given by eq. (20), S^0:$N(4,4)$ = = 700, $N(5,5)$=9 000 , $N(6,6)$=1 000 and zero elsewhere for the more advanced and: S^0: $N(7,7)$=100 , $N(8,8)$=9000 ,

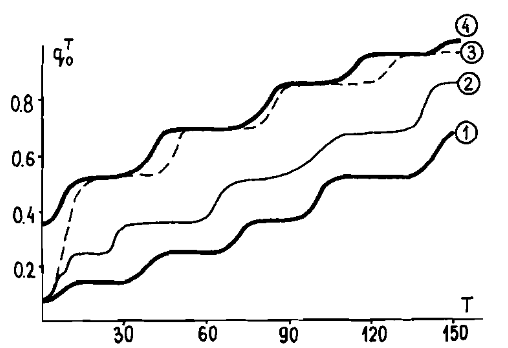

Fig. 15. Pursuit strategies for less advanced generation.

$N(9,9)$=700 and zero elsewhere for less advanced generation. The curves 2 and 3 show what occurred when 4 elements belonging to more advanced generation- respectively of the best and of the just superseded type - have been introduced to the less advanced.
The operation alone and its results seems to be similar to those associated with licence introducing [8].

3.4. Forecasting

A forecasting seems to be one of more promising fields of model applications. It can be carried out under following assumptions:

- a specific environment should be suitably delimited, i.e. expected preferences are the same for elements of the same types - it does not matter that only the best type dominates, and its demand defined.
- a quality function - preferences - does not vary in time or this variations are defined.
- a reproduction process regards the preferences in a fashion assumed.
- a sufficient interval of generation history is known to the extent which make possible to determine the number of all elements involved in the environment demand fulfilling in some previous generations.

It is not necessary to know the parameters of types and quality function shape.
The idea of forecasting is to estimate the relative qualities of particular types knowing the relative ratios of their reproduction in previous generations and to use these qualities to simulate the development assuming none new type will appear. Having sufficient set of such realizations we can determine the prognosis as an average of such realizations as well as it variance. Assuming the quality and time in which the new better type will appear, we can determine the range of prognosis e.g. as a time in which the number of this new type elements will be less then 10% of whole generation number.

To illustrate the problem of generation state forecasting we use a data obtained from simulation carried out under following assumptions: M=N =10 , m =3 10 ,

$$q\left(d_1,d_2\right)=\exp\left(-0.003\left(\left(d_1+d_2\right)^2+\right.\right.$$
$$\left.\left.+16\left(d_2-d_1\right)^2\right)\right) \qquad (22)$$

The initial state is characterised in table 1.

Table 1. Number and quality of elements of particular types in S^0.

type	number	quality	type	number	quality
1	3	0.383	11	2	0.173
2	24	0.444	12	6	0.312
3	238	0.405	13	727	0.323
4	151	0.312	14	2764	0.301
5	1	0.220	15	300	0.258
6	4	0.383	16	2	0.193
7	77	0.405	17	16	0.249
8	2054	0.379	18	128	0.258
9	3293	0.323	19	116	0.234
10	54	0.249	20	1	0.193

Fig. 16. gives the number of elements for some of this types, the new types which appeared as early as in 7-th generation have been also included.

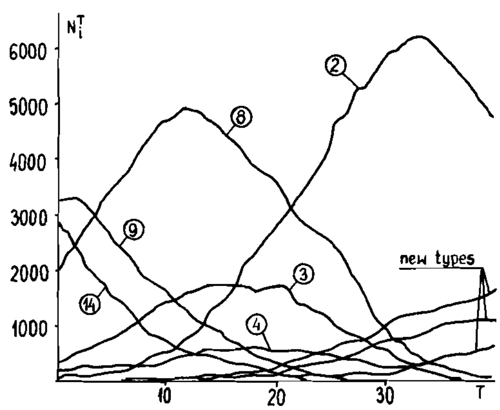

Fig. 16. Elements number for some types in exemplary realization.

We "forecasted" this exemplary realization assuming that only information we have is a number and quality of elements in initial state and that none new type appears. Fig. 17. make possible to compare the exemplary realization, deterministic prognosis and one of random prognosis realization for some types.
The prognosis seems to be quite resonable though more then 1 000 elements of better types existed in 30-generation.

Figs. 18. and 19. yield some information concerning random prognosis convergence. For: M=1 000, N_0^0=100, initial state:

$N_1^0=40$, $N_2^0=30$, $N_3^0=20$, $N_4^0=8$, $N_5^0=2$,
$q_1^0=1.0$, $q_2^0=1.1$, $q_3^0=1.2$, $q_4^0=1.3$ $q_5^0=1.4$,

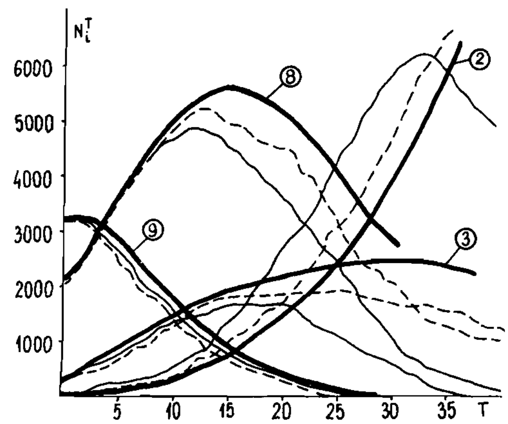

Fig. 17. Forecasting results for types: 2, 3, 8, 9.
—— exemplary realization
—— deterministic prognosis
---- random prognosis realization

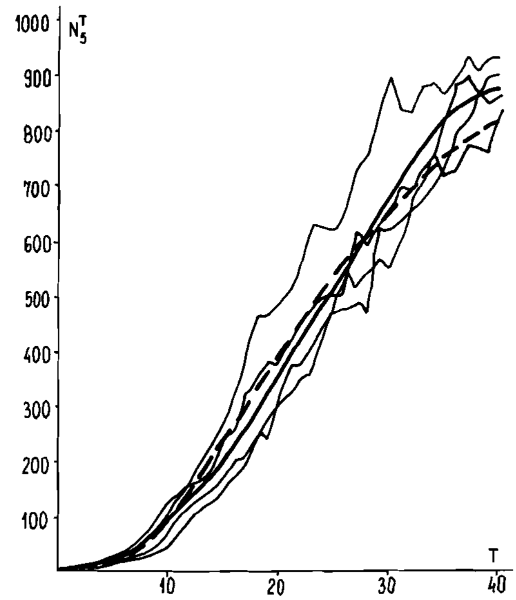

Fig. 18. Number of 5-th type elements.
—— random realization
—— average of realizations
--- deterministic curve

some realizations have been computed. Four of this realization as well as average and deterministic prognosis are drawn for 3-rd and 5-th type respectively.

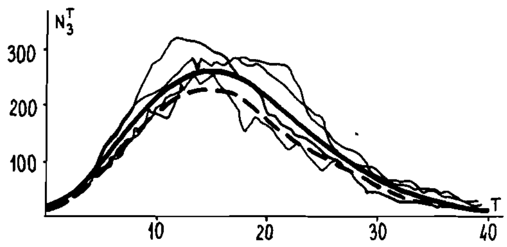

Fig. 19. Number of 3-rd type elements.
—— random realization
—— average of realizations
--- deterministic curve

4. CONCLUSIONS

The model discussed in this paper is entirely abstract, simulation results have been obtained under quite arbitrary assumptions. The same time, the process the model describes manifests quite a lot of properties which are characteristic for some of real life development processes.

We belive there are e.g.

- development itself is a matter of preferences expressed in some environment,
- there are two trends composed: to supersede all elements by temporarily the best ones and to find still better elements,
- irrespectively of significant influence of random factors the growth curves we meet in development processes are very similiar - logistic e.g. (fig. 5.),
- one can trace - usualy a posteriori - a trajectory of development in parameter space, there is a constant growth of quality along this trajectory,
- there are typical stages of development, the stages of transition when new better elements gain domination and the stages of quasi-equilibrium when the new better types of elements are mostly found (fig. 2.),
- there exist always elements of types "deviated" from this temporarily the best, successful modifications are usualy modyfications of this very deviated types,
- the more elements of deviated types exist and the more significant are this deviations the more rapid is a development process (fig. 12.),
- the price of development is a cut in current average quality of population,
- a sudden change in environmental preferences can relase a rapid development figs. 9 and 10. ,
- some directions of development can be missed and it is not easily reversible process fig. 13. ,
- most of succesful innovations are found as a result of recombination of existing types fig. 14. ,
- a contact of isolated populations results usualy in rapid development (fig. 15.).

We hope, the model presented will prove its usability as a forecasting tool and perhaps it can be of some use when the development control means are considered. As for a present, we try to check the model using real life data. The main difficulty is that accesible data are usualy collected rather for specific type in different environments, then for different types in specific environment.

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