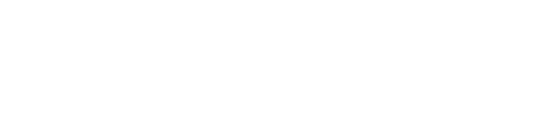
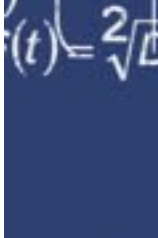


Follow

You have 1 free member-only story left this month. [Sign up for Medium](#) and get an extra one



# The Longest Equations In Physics And Mathematics



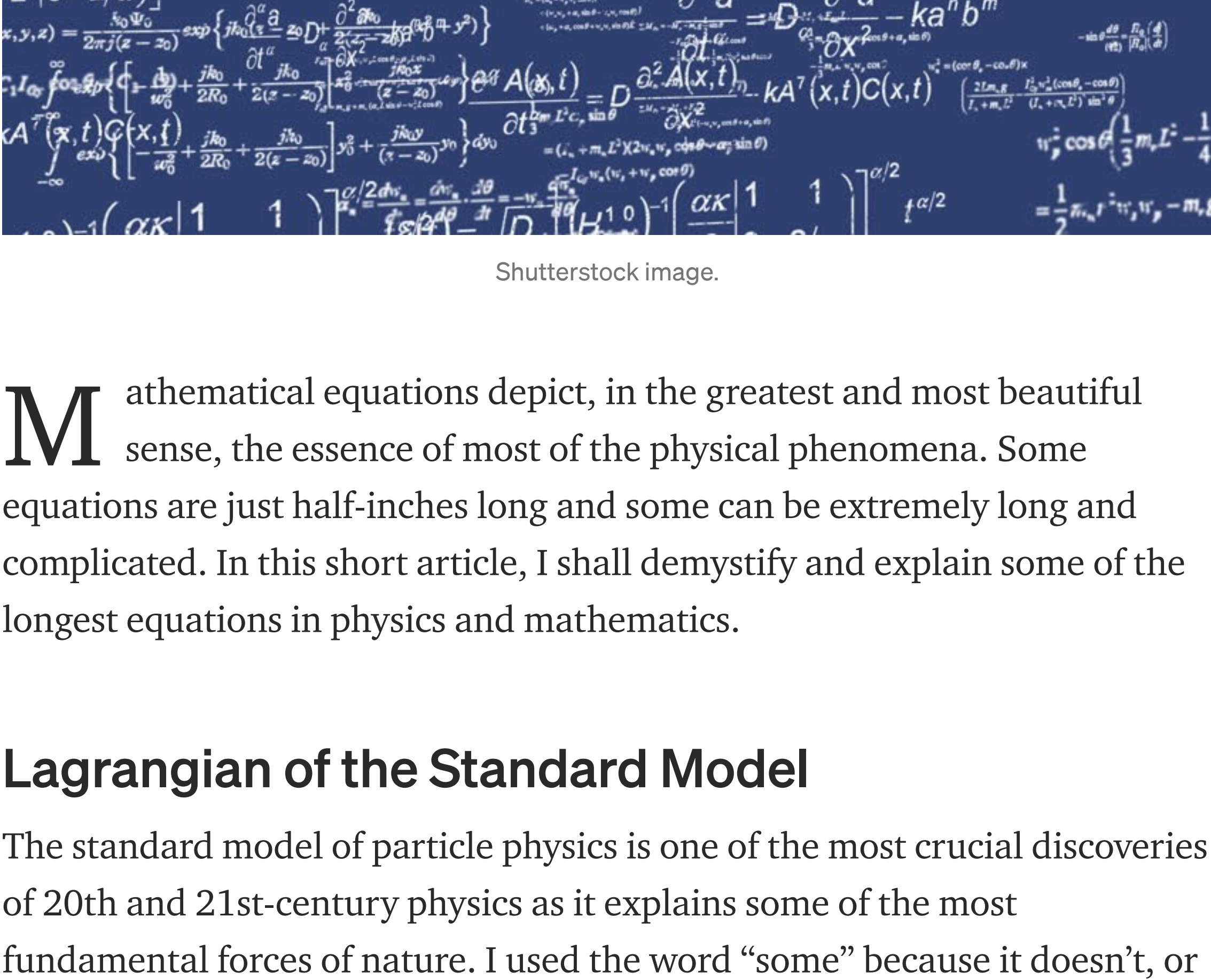
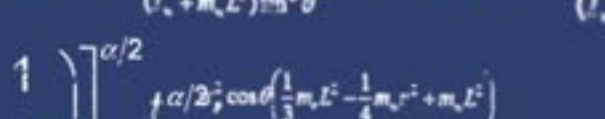
Sunny Labh

Follow

🔒

Jan 2 · 3 min read

★



Shutterstock image.

Mathematical equations depict, in the greatest and most beautiful sense, the essence of most of the physical phenomena. Some equations are just half-inches long and some can be extremely long and complicated. In this short article, I shall demystify and explain some of the longest equations in physics and mathematics.

## Lagrangian of the Standard Model

The standard model of particle physics is one of the most crucial discoveries of 20th and 21st-century physics as it explains some of the most fundamental forces of nature. I used the word “some” because it doesn’t, or so far hasn’t been able to fully, explain the weakest force of all- gravity. The model can be represented in many different ways. You might be familiar with the *periodic table-like* representation where the particles are arranged in a specific manner. It can also be represented mathematically in different forms. However, one such form exists which explains it in quite an interesting way, *the Lagrangian form*. The Lagrangian is a fancy way of writing an equation to determine the state of a changing system and explain the maximum possible energy the system can maintain. It is one of the most compact ways of explaining the standard model.

$$\begin{aligned} \mathcal{L}_{SM} = & -\frac{1}{2}\partial_\mu g_\mu^a \partial_\nu g_\mu^a - g_s f^{abc} \partial_\mu g_\mu^a g_\nu^b g_\mu^c - \frac{1}{2}g_s^2 f^{abc} f^{ade} g_\mu^b g_\nu^d g_\mu^e g_\mu^f - \partial_\mu W_\mu^+ \partial_\nu W_\mu^- - \\ & M^2 W_\mu^+ W_\mu^- - \frac{1}{2}\partial_\mu Z_\mu^0 \partial_\nu Z_\mu^0 - \frac{1}{2}M_Z^2 Z_\mu^0 Z_\mu^0 - \frac{1}{2}\partial_\mu A_\mu \partial_\nu A_\nu - ig_{cw}(\partial_\mu Z_\mu^0(W_\mu^+ W_\mu^- - \\ & W_\mu^- W_\mu^-) - Z_\mu^0(W_\mu^+ \partial_\mu W_\mu^- - W_\mu^- \partial_\mu W_\mu^+) + Z_\mu^0(W_\mu^+ \partial_\mu W_\mu^- - W_\mu^- \partial_\mu W_\mu^-) - \\ & ig_{sw}(\partial_\mu A_\mu(W_\mu^+ W_\mu^- - W_\mu^- W_\mu^-) - A_\mu(W_\mu^+ \partial_\mu W_\mu^- - W_\mu^- \partial_\mu W_\mu^+) + A_\mu(W_\mu^+ \partial_\mu W_\mu^- - \\ & W_\mu^- \partial_\mu W_\mu^+)) - \frac{1}{2}g^2 W_\mu^+ W_\mu^- W_\mu^+ W_\mu^- + \frac{1}{2}g^2 W_\mu^+ W_\mu^- W_\mu^- W_\mu^- + g^2 c_w^2 (Z_\mu^0 W_\mu^+ Z_\mu^0 W_\mu^- - \\ & Z_\mu^0 Z_\mu^0 W_\mu^+ W_\mu^-) + g^2 s_w^2 (A_\mu W_\mu^+ A_\mu W_\mu^- - A_\mu A_\mu W_\mu^+ W_\mu^-) + g^2 s_w c_w (A_\mu Z_\mu^0 W_\mu^+ W_\mu^- - \\ & W_\mu^- W_\mu^-) - 2A_\mu Z_\mu^0 W_\mu^+ W_\mu^-) - \frac{1}{2}\partial_\mu H \partial_\nu H - 2M^2 \alpha_h H^2 - \partial_\mu \phi^+ \partial_\nu \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\nu \phi^0 - \\ & \beta_h \left( \frac{2M_h^2}{g^2} + \frac{2M_h}{g} H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-) \right) + \frac{2M_h^4}{g^2} \alpha_h - \\ & \frac{1}{8}g^2 \alpha_h (H^4 + (\phi^0)^4 + 4(\phi^0 \phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4H^2 \phi^+ \phi^- + 2(\phi^0)^2 H^2) - \\ & g M W_\mu^+ W_\mu^- H - \frac{1}{2}g \frac{M_h^2}{g^2} Z_\mu^0 Z_\mu^0 H - \\ & \frac{1}{2}ig (W_\mu^+ (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - W_\mu^- (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)) + \\ & \frac{1}{2}g (W_\mu^+ (H \partial_\mu \phi^- - \phi^- \partial_\mu H) + W_\mu^- (H \partial_\mu \phi^+ - \phi^+ \partial_\mu H)) + \frac{1}{2}g \frac{1}{c_w} (Z_\mu^0 (H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) + \\ & M (\frac{1}{c_w} Z_\mu^0 \partial_\mu \phi^0 + W_\mu^+ \partial_\mu \phi^- + W_\mu^- \partial_\mu \phi^+) - ig \frac{M_h^2}{c_w} Z_\mu^0 (W_\mu^+ \phi^- - W_\mu^- \phi^+) + ig s_w M A_\mu (W_\mu^+ \phi^- - \\ & W_\mu^- \phi^+) - ig \frac{1}{2} \frac{2M_h^2}{c_w} Z_\mu^0 (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + ig s_w A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \\ & \frac{1}{2}g^2 W_\mu^+ W_\mu^- (H^2 + (\phi^0)^2 + 2\phi^+ \phi^-) - \frac{1}{2}g^2 \frac{M_h^2}{c_w} Z_\mu^0 (H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)\phi^+ \phi^-) - \\ & \frac{1}{2}g^2 \frac{M_h^2}{c_w} Z_\mu^0 \phi^0 (W_\mu^+ \phi^- + W_\mu^- \phi^+) - \frac{1}{2}ig^2 \frac{M_h^2}{c_w} Z_\mu^0 H (W_\mu^+ \phi^- - W_\mu^- \phi^+) + \frac{1}{2}g^2 s_w A_\mu \phi^0 (W_\mu^+ \phi^- + \\ & W_\mu^- \phi^+) + \frac{1}{2}ig^2 s_w A_\mu H (W_\mu^+ \phi^- - W_\mu^- \phi^+) - g^2 \frac{M_h^2}{c_w} (2c_w^2 - 1) Z_\mu^0 A_\mu \phi^+ \phi^- - \\ & g^2 s_w^2 A_\mu A_\mu \phi^+ \phi^- + \frac{1}{2}ig_s \lambda_g^2 (\bar{q}^i \gamma^\mu q^j) q_\mu^i - e^2 (\gamma^\mu + m_\ell^2) e^\lambda - \nu^\lambda (\gamma^\mu + m_\ell^2) \nu^\lambda - u_\ell^2 (\gamma^\mu + \\ & m_\ell^2) u_\ell^2 - d_\ell^2 (\gamma^\mu + m_\ell^2) d_\ell^2 + ig s_w A_\mu (- (e^i \gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}_j^i \gamma^\mu u_j^\lambda) - \frac{1}{3}(\bar{d}_j^i \gamma^\mu d_j^\lambda)) + \\ & \frac{ig}{4c_w} Z_\mu^0 \{ (\bar{\nu}^i \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (e^i \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{d}_j^i \gamma^\mu (\frac{2}{3}s_w^2 - 1 - \gamma^5) d_j^\lambda) + \\ & (\bar{u}_j^i \gamma^\mu (1 - \frac{2}{3}s_w^2 + \gamma^5) u_j^\lambda) \} + \frac{ig}{2\sqrt{2}} W_\mu^+ ((\bar{\nu}^i \gamma^\mu (1 + \gamma^5) U^{i\mu} e^\lambda) + (\bar{u}_j^i \gamma^\mu (1 + \gamma^5) C_{\lambda\mu} d_j^\lambda)) + \\ & \frac{ig}{2\sqrt{2}} W_\mu^- ((\bar{e}^i U^{i\mu} \nu_\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{d}_j^i C_{\lambda\mu} \gamma^\mu (1 + \gamma^5) u_j^\lambda)) + \\ & \frac{ig}{2M\sqrt{2}} \phi^+ (-m_\ell^2 (\bar{\nu}^i \nu^\lambda U^{i\mu} e^\lambda (1 - \gamma^5) e^\lambda) + m_\ell^2 (\bar{\nu}^i \nu^\lambda U^{i\mu} e^\lambda (1 + \gamma^5) e^\lambda) + \\ & \frac{ig}{2M\sqrt{2}} \phi^- (m_\ell^2 (\bar{e}^i U^{i\mu} \nu_\lambda (1 + \gamma^5) \nu^\lambda) - m_\ell^2 (\bar{e}^i U^{i\mu} \nu_\lambda (1 - \gamma^5) \nu^\lambda) - \frac{g}{2} \frac{m_\ell^2}{M} H (\bar{\nu}^i \nu^\lambda) - \\ & \frac{g}{2} \frac{m_\ell^2}{M} H (\bar{e}^i e^\lambda) + \frac{ig}{2} \frac{m_\ell^2}{M} \phi^0 (\bar{\nu}^i \gamma^5 \nu^\lambda) - \frac{ig}{2} \frac{m_\ell^2}{M} \phi^0 (\bar{e}^i \gamma^5 e^\lambda) - \frac{1}{4} \bar{\nu}_\lambda M_\ell^R (1 - \gamma_5) \bar{\nu}_\mu - \\ & \frac{1}{4} \bar{\nu}_\lambda M_\ell^R (1 - \gamma_5) \bar{\nu}_\mu + \frac{ig}{2M\sqrt{2}} \phi^+ (-m_\ell^2 (\bar{u}_j^i C_{\lambda\mu} (1 - \gamma^5) d_j^\lambda) + m_\ell^2 (\bar{u}_j^i C_{\lambda\mu} (1 + \gamma^5) d_j^\lambda) + \\ & \frac{ig}{2M\sqrt{2}} \phi^- (m_\ell^2 (\bar{d}_j^i C_{\lambda\mu} (1 + \gamma^5) u_j^\lambda) - m_\ell^2 (\bar{d}_j^i C_{\lambda\mu} (1 - \gamma^5) u_j^\lambda) - \frac{g}{2} \frac{m_\ell^2}{M} H (\bar{u}_j^i u_j^\lambda) - \\ & \frac{g}{2} \frac{m_\ell^2}{M} H (\bar{d}_j^i d_j^\lambda) + \frac{ig}{2} \frac{m_\ell^2}{M} \phi^0 (\bar{u}_j^i \gamma^5 u_j^\lambda) - \frac{ig}{2} \frac{m_\ell^2}{M} \phi^0 (\bar{d}_j^i \gamma^5 d_j^\lambda) + \bar{G}^a \partial^2 G^a + g_s f^{abc} \partial_\mu \bar{G}^a G^a \partial_\mu g_\mu^c + \\ & \bar{X}^+ (\partial^2 - M^2) X^- + \bar{X}^- (\partial^2 - M^2) X^+ + \bar{X}^0 (\partial^2 - \frac{M_X^2}{c_w^2}) X^0 + \bar{Y} \partial^2 Y + ig_{cw} W_\mu^+ (\partial_\mu \bar{X}^0 X^- - \\ & \partial_\mu \bar{X}^+ X^0) + ig s_w W_\mu^+ (\partial_\mu \bar{Y} X^- - \partial_\mu \bar{X}^+ Y) + ig c_w W_\mu^- (\partial_\mu \bar{X}^- X^0 - \\ & \partial_\mu \bar{X}^0 X^+) + ig s_w W_\mu^- (\partial_\mu \bar{X}^- Y - \partial_\mu \bar{Y} X^+) + ig c_w Z_\mu^0 (\partial_\mu \bar{X}^+ X^- - \\ & \partial_\mu \bar{X}^- X^+) + ig s_w A_\mu (\partial_\mu \bar{X}^+ X^- - \\ & \partial_\mu \bar{X}^- X^+) - \frac{1}{2}g M (\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{2} \bar{X}^0 X^0 H) + \frac{1}{2\sqrt{2}} ig M (\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-) + \\ & \frac{1}{2c_w} ig M (\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-) + ig M s_w (\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-) + \\ & \frac{1}{2} ig M (\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0) . \end{aligned}$$

Lagrangian form of the standard model. Thomas Gutierrez, an assistant professor of Physics at California Polytechnic State University, transcribed the Standard Model Lagrangian for the web. He derived it from Diagrammatica, a theoretical physics reference written by Nobel Laureate Martinus Veltman.

The story of the Standard Model started in the 1960s with the elaboration of the theory of quarks and leptons, and continued for about five decades until the discovery of the Higgs boson in 2012.

Explicitly, the parts forming the entire Lagrangian generally consist of :  
**Free fields: massive vector bosons, photons, and leptons.**  
**Fermion fields describing matter.**  
**The Lepton-boson interaction.**  
**Third-order and fourth-order interactions of vector bosons.**  
**The Higgs section.**

## The Quadratic Formula

We all are familiar with the general second-degree polynomial quadratic formula with provides a solution to the equation under consideration. The cubic formula, for a third-degree polynomial, is even longer, despite still being of modest size and certainly within reason to memorize.

$$\begin{aligned} x = & \frac{3}{\sqrt{\frac{-b^3}{27a^3} - \frac{bc}{6a^2} - \frac{d}{2a}}} + \sqrt{\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}}^2 + \left(\frac{c}{3a} - \frac{b^2}{6a^2}\right)^3 \\ & + \frac{3}{\sqrt{\frac{-b^3}{27a^3} - \frac{bc}{6a^2} - \frac{d}{2a}}} - \sqrt{\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}}^2 + \left(\frac{c}{3a} - \frac{b^2}{6a^2}\right)^3 - \frac{b}{3a} . \end{aligned}$$

Formula for the solution of third-degree polynomial

The formula for the solution of a fourth-degree polynomial, however, is truly massive, though not much complicated.

The **solution to a general quintic equation in terms of hypergeometric functions** by the *Bring-Jerrard reduction* might be a good candidate. A paper titled [ON THE COMPLETE SOLUTION TO THE MOST GENERAL FIFTH DEGREE POLYNOMIAL](#) by Richard J. Drociuk at the Physics Department of Simon Fraser University provides e closed-form solution for the five roots of the General Quintic Equation. The paper has at the end some of the equations in computer notation but not plugged together. When plugged together, they expand to form the full equation at *large asteroid* size.

The longest math equation contains around 200 terabytes of text called [the Boolean Pythagorean Triples problem](#). It was first proposed by California-based mathematician Ronald Graham, back in the 1980s.

...

Thank you so much for reading. If you like my work and want to support me then please sign up to become a **medium member using [this link](#)** or else, you can **[buy me a coffee](#)** ☕.

Physics

Mathematics

Life

Education

Learning

131

WRITTEN BY

Sunny Labh

Science writer majoring in quantum mechanics. Writer and Editor at @cantor\_paradise | Founder @PhysInHistory | Email: sunnylabh11@gmail.com

Follow

🔒

Cantor's Paradise

Medium's #1 Math Publication

Follow

### More From Medium

Branch and bound

ADDA TIRU VENKATA SAI TARUN

Permutation—Order Matter!

Suraj Singh in Analytics Vidhya

Top 5 books to learn & understand math from the basics

Martin Kondor in Intuition

How To Find Domain & Range -Part-II

बबीता गुप्ता

Machine Learning Definitions

David Cittadini

Karen Uhlenbeck Won the 'Nobel of Math'—but Women Are Still Under-Represented in the Field

Popular Science in Popular Science

Let's keep this discrete.

Gabrielle Birchak in MathScienceHistory

The Problem with Causality

Nuwan I. Senarathna in On Philosophy

Learn more.

Medium is an open platform where 170 million readers come to find insightful and dynamic thinking. Here, expert and undiscovered voices alike dive into the heart of any topic and bring new ideas to the surface. [Learn more](#)

Make Medium yours.

Follow the writers, publications, and topics that matter to you, and you'll see them on your homepage and in your inbox. [Explore](#)

Write a story on Medium.

If you have a story to tell, knowledge to share, or a perspective to offer — welcome home. It's easy and free to post your thinking on any topic. [Start a blog](#)