



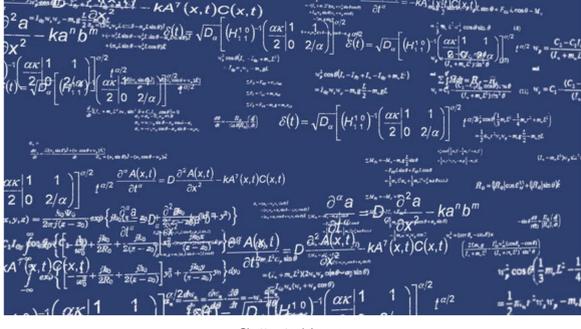
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The Longest Equations In Physics And Mathematics

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Mathematical equations depict, in the greatest and most beautiful sense, the essence of most of the physical phenomena. Some equations are just half-inches long and some can be extremely long and complicated. In this short article, I shall demystify and explain some of the longest equations in physics and mathematics.

Lagrangian of the Standard Model

The standard model of particle physics is one of the most crucial discoveries of 20th and 21st-century physics as it explains some of the most fundamental forces of nature. I used the word "some" because it doesn't, or so far hasn't been able to fully, explain the weakest force of all - gravity. The model can be represented in many different ways. You might be familiar with the periodic table-like representation where the particles are arranged in a specific manner. It can also be represented mathematically in different forms. However, one such form exists which explains it in quite an interesting way, the Lagrangian form. The Lagrangian is a fancy way of writing an equation to determine the state of a changing system and explain the maximum possible energy the system can maintain. It is one of the most compact ways of explaining the standard model.

$$\mathcal{L}_{SM} = -\frac{1}{4}g_s^2 G_{\mu\nu}^a G^{\mu\nu a} - g_s f^{abc} \partial_\mu g_\nu^a g_\rho^b g_\sigma^c - \frac{1}{4}g_w^2 f^{abc} f^{def} g_\mu^a g_\nu^b g_\rho^c g_\sigma^d - \partial_\mu W_\nu^+ \partial_\mu W_\nu^- - M^2 W_\mu^+ W_\mu^- - \frac{1}{2}\partial_\mu Z_\nu^0 \partial_\mu Z_\nu^0 - \frac{1}{2}M_Z^2 Z_\mu^0 Z_\mu^0 - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - ig_{cw}(\partial_\mu Z_\nu^0 W_\mu^+ W_\nu^- - W_\mu^+ \partial_\nu W_\nu^- - W_\mu^- \partial_\nu W_\nu^+) + Z_\nu^0(W_\mu^+ \partial_\nu W_\nu^- - W_\mu^- \partial_\nu W_\nu^+) - ig_{sw}(\partial_\mu A_\nu W_\mu^+ W_\nu^- - W_\mu^+ \partial_\nu W_\nu^-) - A_\mu(W_\mu^+ \partial_\nu W_\nu^- - W_\mu^- \partial_\nu W_\nu^+) + A_\mu(W_\mu^+ \partial_\nu W_\nu^- - W_\mu^- \partial_\nu W_\nu^+) - \frac{1}{2}\phi^\dagger W_\mu^+ W_\mu^- \phi + \frac{1}{2}\phi^\dagger W_\mu^+ W_\mu^- \phi + g^2 c_w^2 (Z_\nu^0 W_\mu^+ Z_\nu^0 W_\mu^- - Z_\nu^0 Z_\nu^0 W_\mu^+ W_\mu^-) + g^2 s_w^2 (A_\mu W_\mu^+ A_\mu W_\mu^- - A_\mu A_\mu W_\mu^+ W_\mu^-) + g^2 s_w c_w (A_\mu Z_\nu^0 W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - 2A_\mu Z_\nu^0 W_\mu^+ W_\nu^- - \frac{1}{2}\partial_\mu H \partial_\mu H - 2M^2 \alpha_h H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - \frac{1}{2}g_\phi^2 \partial_\mu \phi^+ \partial_\mu \phi^- - \beta_h \left(\frac{2M^2}{g^2} + \frac{2M}{g} H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-) \right) + \frac{2M^4}{g^2} \alpha_h - \frac{1}{2}g^2 \alpha_h (H^4 + (\phi^0)^4 + 4(\phi^0)^2 \phi^+ \phi^- + 4(H^2 \phi^+ \phi^- + 2(\phi^0)^2 H^2) - g M W_\mu^+ W_\mu^- H - \frac{1}{2}g \frac{M^2}{g^2} Z_\nu^0 Z_\nu^0 H - \frac{1}{2}ig (W_\mu^+ (\partial^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - W_\mu^- (\partial^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)) + \frac{1}{2}g (W_\mu^+ (H \partial_\mu \phi^- - \phi^- \partial_\mu H) + W_\mu^- (H \partial_\mu \phi^+ - \phi^+ \partial_\mu H)) + \frac{1}{2}g \frac{M^2}{g^2} (H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) + M \left(\frac{1}{2} Z_\nu^0 \partial_\mu \phi^0 + W_\mu^+ \partial_\nu \phi^- + W_\mu^- \partial_\nu \phi^+ \right) - ig \frac{M^2}{g^2} M Z_\nu^0 (W_\mu^+ \phi^- - W_\mu^- \phi^+) + ig s_w M A_\mu (W_\mu^+ \phi^- - W_\mu^- \phi^+) - ig \frac{1}{2} \frac{2M^2}{2g c_w} Z_\nu^0 (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + ig s_w A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \frac{1}{2}g^2 W_\mu^+ W_\mu^- (H^2 + (\phi^0)^2 + 2\phi^+ \phi^-) - \frac{1}{2}g^2 Z_\nu^0 Z_\nu^0 (H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)\phi^+ \phi^-) - \frac{1}{2}g^2 c_w^2 Z_\nu^0 \phi^0 (W_\mu^+ \phi^- + W_\mu^- \phi^+) - \frac{1}{2}ig^2 s_w^2 Z_\nu^0 H (W_\mu^+ \phi^- - W_\mu^- \phi^+) + \frac{1}{2}g^2 s_w A_\mu \phi^0 (W_\mu^+ \phi^- - W_\mu^- \phi^+) + \frac{1}{2}ig^2 s_w A_\mu H (W_\mu^+ \phi^- - W_\mu^- \phi^+) - g^2 c_w^2 (2c_w^2 - 1) Z_\nu^0 A_\mu \phi^0 \phi^- - g^2 s_w^2 A_\mu A_\mu \phi^+ \phi^- + \frac{1}{2}ig^2 s_w A_\mu H (W_\mu^+ \phi^- - W_\mu^- \phi^+) - e^2 (\gamma\theta + m_\nu^2) e^2 - \nu^2 (\gamma\theta + m_\nu^2) \nu^2 - u_\nu^2 (\gamma\theta + m_\nu^2) u_\nu^2 - d_\nu^2 (\gamma\theta + m_\nu^2) d_\nu^2 + ig s_w A_\mu (-e^2 \gamma^\mu e^2) + \frac{3}{2}(u_\nu^2 \gamma^\mu u_\nu^2) - \frac{1}{2}(d_\nu^2 \gamma^\mu d_\nu^2) + \frac{M^2}{2c_w} Z_\nu^0 ((\nu^2 \gamma^\mu (1 + \gamma^5)) \nu^2) + (e^2 \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^2) + (d_\nu^2 \gamma^\mu (\frac{2}{3}s_w^2 - 1 - \gamma^5) d_\nu^2) + (u_\nu^2 \gamma^\mu (1 - \frac{2}{3}s_w^2 + \gamma^5) u_\nu^2) + \frac{ig}{2} W_\mu^+ ((\nu^2 \gamma^\mu (1 + \gamma^5) U^{lep} \nu_e) + (u_\nu^2 \gamma^\mu (1 + \gamma^5) C_{\nu e} d_\nu^2)) + \frac{2ig}{2M} W_\mu^- ((e^2 U^{lep} \nu_e \gamma^\mu (1 + \gamma^5) \nu^2) + (d_\nu^2 C_{\nu e} \gamma^\mu (1 + \gamma^5) u_\nu^2)) + \frac{ig}{2M} W_\mu^- (-m_\nu^2 (\nu^2 U^{lep} \nu_e (1 - \gamma^5) e^2) + m_\nu^2 (\nu^2 U^{lep} \nu_e (1 + \gamma^5) e^2) + \frac{ig}{2M} W_\mu^- (m_\nu^2 (e^2 U^{lep} \nu_e (1 + \gamma^5) \nu^2) - m_\nu^2 (e^2 U^{lep} \nu_e (1 - \gamma^5) \nu^2) - \frac{g}{2} \frac{M^2}{M} H (\nu^2 \nu^2) - \frac{g}{2} \frac{M^2}{M} H (e^2 e^2) + \frac{ig}{2} \frac{M^2}{M} \phi^0 (\nu^2 \gamma^5 \nu^2) - \frac{ig}{2} \frac{M^2}{M} \phi^0 (e^2 \gamma^5 e^2) - \frac{1}{4} \nu_\lambda M_\lambda^R (1 - \gamma_5) \nu_\lambda - \frac{1}{4} \nu_\lambda M_\lambda^R (1 - \gamma_5) \nu_\lambda + \frac{ig}{2M} \phi^0 (-m_\nu^2 (u_\nu^2 C_{\nu e} (1 - \gamma^5) d_\nu^2) + m_\nu^2 (u_\nu^2 C_{\nu e} (1 + \gamma^5) d_\nu^2) + \frac{1}{2} \nu_\lambda M_\lambda^R (1 - \gamma_5) \nu_\lambda) - m_\nu^2 (d_\nu^2 C_{\nu e} (1 - \gamma^5) u_\nu^2) - \frac{g}{2} \frac{M^2}{M} H (u_\nu^2 u_\nu^2) - \frac{g}{2} \frac{M^2}{M} H (d_\nu^2 d_\nu^2) + \frac{ig}{2} \frac{M^2}{M} \phi^0 (u_\nu^2 \gamma^5 u_\nu^2) - \frac{ig}{2} \frac{M^2}{M} \phi^0 (d_\nu^2 \gamma^5 d_\nu^2) + G^a \partial^2 G^a + g_s f^{abc} \partial_\mu G^a G^b G^c + \bar{X}^\dagger (\partial^2 - M^2) X^\dagger + \bar{X}^\dagger (\partial^2 - M^2) X^\dagger + \bar{X}^\dagger \partial^2 X^\dagger + \bar{Y} \partial^2 Y + ig_{cw} W_\mu^+ (\partial_\mu \bar{X} X^0 - \partial_\mu X^0 X^+) + ig_{sw} W_\mu^+ (\partial_\mu \bar{X} X^- - \partial_\mu X^- X^+) + ig_{cw} W_\mu^- (\partial_\mu \bar{X} X^0 - \partial_\mu X^0 X^-) + ig_{sw} W_\mu^- (\partial_\mu \bar{X} X^- - \partial_\mu X^- X^+) + ig_{cw} Z_\nu^0 (\partial_\mu \bar{X} X^+ - \partial_\mu X^+ X^-) + ig_{sw} Z_\nu^0 (\partial_\mu \bar{X} X^- - \partial_\mu X^- X^+) - \frac{1}{2}igM (\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{2}\bar{X}^0 X^0 H) + \frac{1}{2}igM (\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-) + \frac{1}{2}igM (\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-) + igM s_w (\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-) + \frac{1}{2}igM (\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0).$$

Lagrangian form of the standard model. Thomas Gutierrez, an assistant professor of Physics at California Polytechnic State University, transcribed the Standard Model Lagrangian for the web. He derived it from Diagrammatica, a theoretical physics reference written by Nobel Laureate Martinus Veltman.

The story of the Standard Model started in the 1960s with the elaboration of the theory of quarks and leptons, and continued for about five decades until the discovery of the Higgs boson in 2012.

Explicitly, the parts forming the entire Lagrangian generally consist of :
Free fields: massive vector bosons, photons, and leptons.
Fermion fields describing matter.
The Lepton-boson interaction.
Third-order and fourth-order interactions of vector bosons.
The Higgs section.

The first three lines of the equation are ultra-specific to the gluons, the boson that carries the strong force. Almost half of this equation is dedicated to explaining interactions between bosons, particularly W and Z bosons. Bosons are force-carrying particles, and there are four species of bosons that interact with other particles using three fundamental forces. The rest half of the equation explains how elementary matter particles interact with the weak force and how matter particles interact with Higgs ghosts (virtual artifacts from the Higgs field).

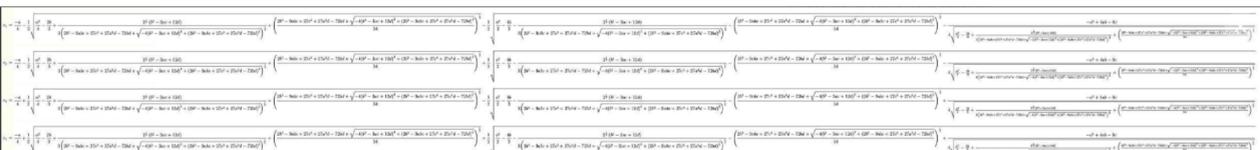
The Quadratic Formula

We all are familiar with the general second-degree polynomial quadratic formula which provides a solution to the equation under consideration. The cubic formula, for a third-degree polynomial, is even longer, despite still being of modest size and certainly within reason to memorize.

$$x = \frac{3 \sqrt{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)} + \sqrt{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3}}{2} + \frac{3 \sqrt{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)} - \sqrt{\left(\frac{-b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3}}{2} - \frac{b}{3a}$$

Formula for the solution of third-degree polynomial

The formula for the solution of a fourth-degree polynomial, however, is truly massive, though not much complicated.



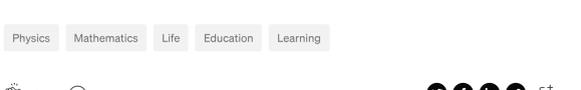
Formula for the solution of a quartic polynomial

The solution to a general quintic equation in terms of hypergeometric functions by the Bring-Jerrard reduction might be a good candidate. A paper titled ON THE COMPLETE SOLUTION TO THE MOST GENERAL FIFTH DEGREE POLYNOMIAL by Richard J. Drociuk at the Physics Department of Simon Fraser University provides a closed-form solution for the five roots of the General Quintic Equation. The paper has at the end some of the equations in computer notation but not plugged together. When plugged together, they expand to form the full equation at large asteroid size.

The longest math equation contains around 200 terabytes of text called the Boolean Pythagorean Triples problem. It was first proposed by California-based mathematician Ronald Graham, back in the 1980s.



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